Quantum electrodynamics of photonic and of absorbing dielectric media

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LSSF in honor of
Leendert Suttorp’s 65th birthday,
Amsterdam, September 23th 2005

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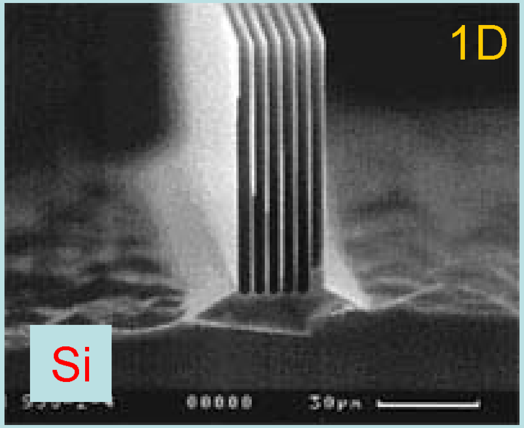
Outline

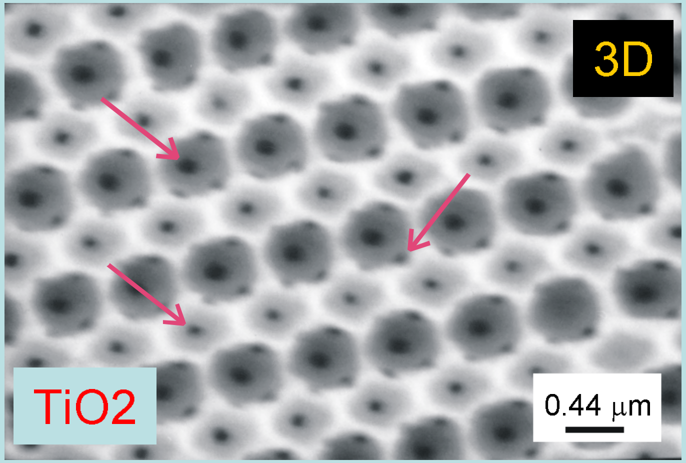
Work done with Leendert Suttorp and Ad Lagendijk

- Part I: Light emission by atoms in photonic media (simple models)
- Part II: Quantum electrodynamics in absorbing dielectrics (formalism)

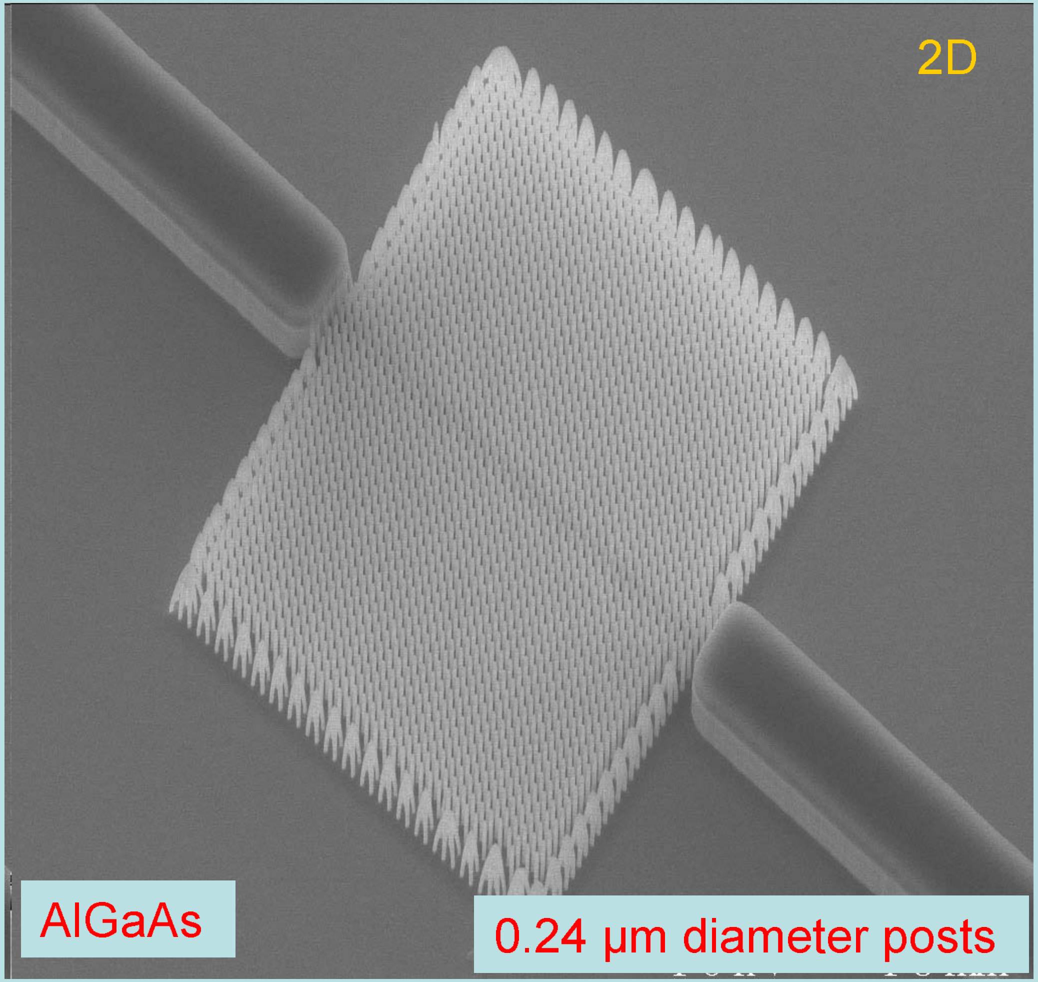
1D, 2D, and 3D photonic crystals



MTG Dublin, 2003



COPS, Twente, 2003



Photonic lattice team, Sandia National Laboratories, 2004

Scattered and bound waves in non-absorbing media

Two types of solutions of wave equation:

1. Scattered (plane) waves

$$|\mathbf{f}_{\mathbf{k}}\rangle = |\mathbf{k}\rangle + \int g_0 V |\mathbf{f}_{\mathbf{k}}\rangle$$

$$\downarrow$$

$$|\mathbf{f}_{\mathbf{k}}\rangle = |\mathbf{k}\rangle + \int g_0 T |\mathbf{k}\rangle$$

2. Bound waves

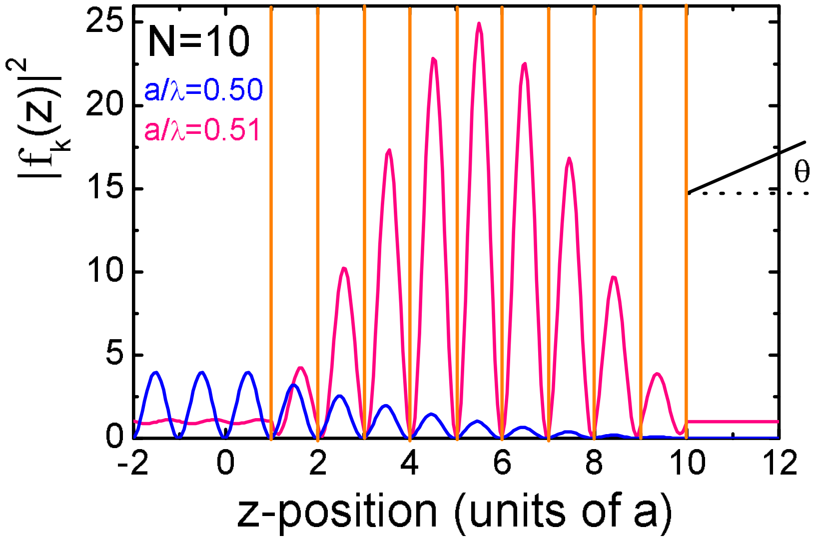
$$|\mathbf{f}_{\lambda}\rangle = \int g_0 V |\mathbf{f}_{\lambda}\rangle$$

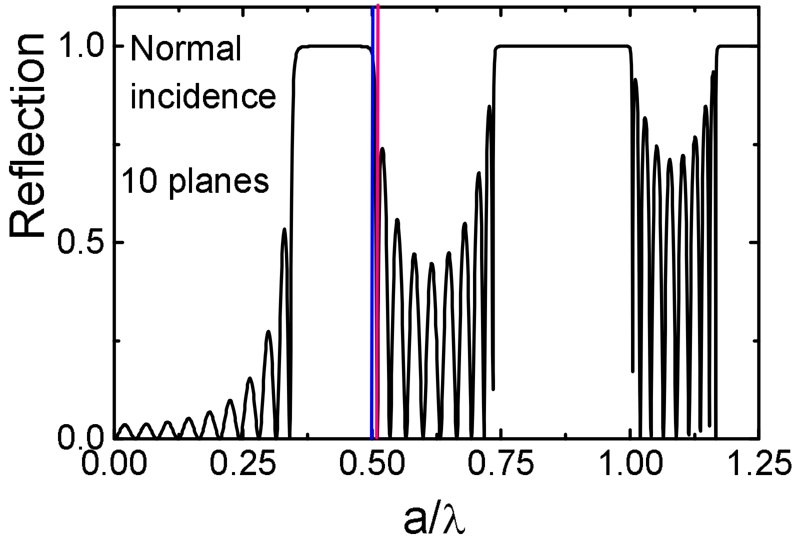
$$\downarrow$$

$$\Rightarrow T^{-1} = 0$$

Crystal of planes: scattered plane waves

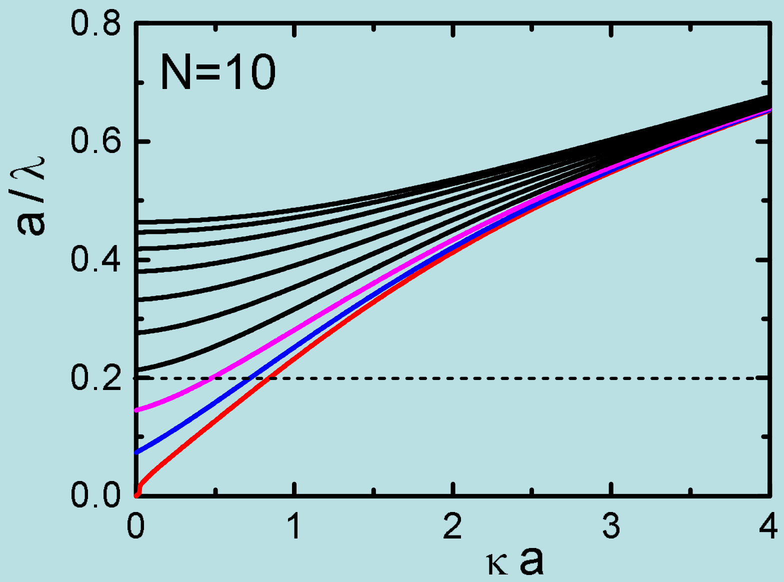
$$\mathbf{f}_{\mathbf{k}}(z) = e^{ik_z z} - \frac{i}{2k_z} \sum_{m,n=1}^{N} T_{mn}^{(N)}(\theta,\omega) e^{2\pi i(a/\lambda)(|z-m|+n)\cos\theta}$$



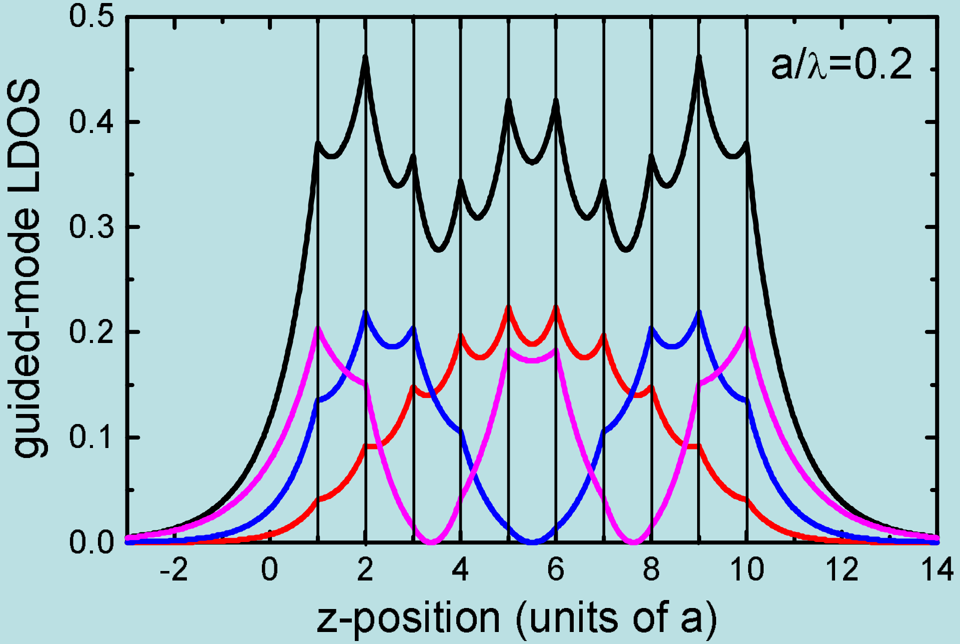


Guided modes of crystal of planes

$$\kappa = \sqrt{k_\parallel^2 - \omega^2/c^2}$$



N planes $\Rightarrow \leq N$ guided modes



$$LDOS(\omega) \propto \mathrm{Im}\, g(\mathbf{r}, \mathbf{r}, \omega)$$

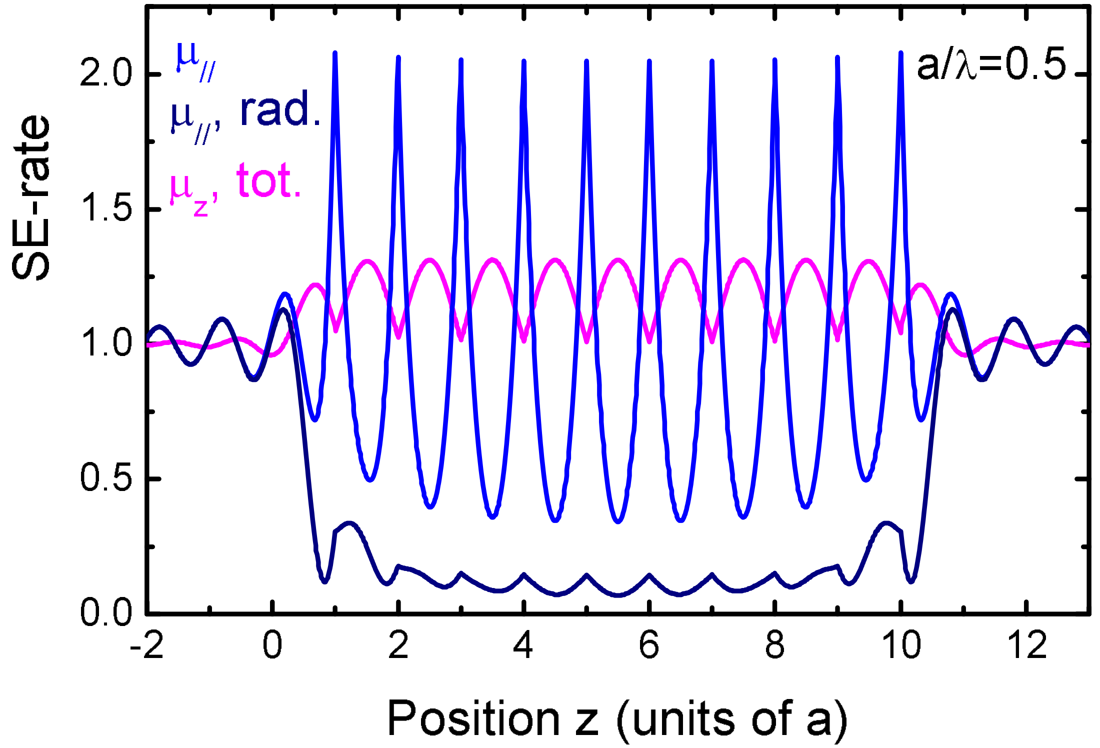
$$= \mathrm{Im} \int \mathrm{d}^2\mathbf{k}_\parallel g(\mathbf{k}_\parallel, z, z, \omega)$$

Wubs & Lagendijk, PRE **65**, 046612 (2002). Aknowledgment: **Leendert Suttorp**

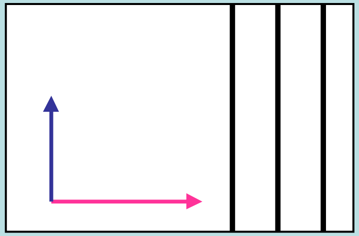
Spontaneous emission in crystal

$$\Gamma(\boldsymbol{\mu},\mathbf{r},\omega) = \frac{-2\omega^2}{\hbar\varepsilon_0 c^2}\,\mathrm{Im}\,[\boldsymbol{\mu}\cdot\mathbf{G}(\mathbf{r},\mathbf{r},\omega)\cdot\boldsymbol{\mu}]$$

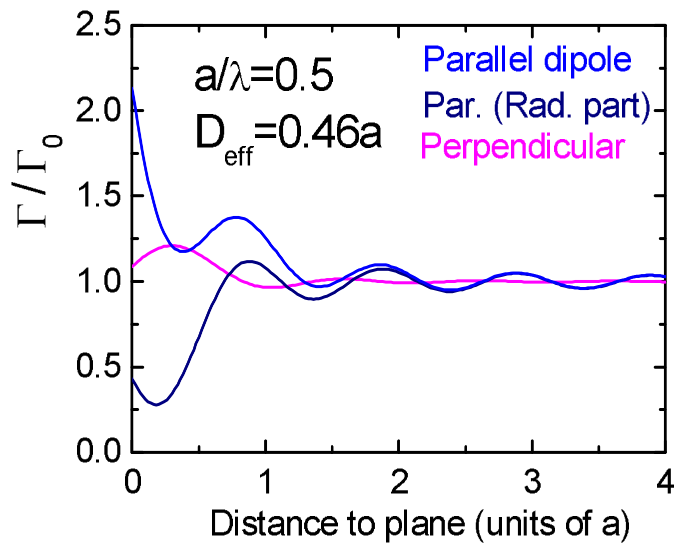
$$\mathbf{G}(\mathbf{k}_{||},z_A,z_B) = \mathbf{G}_0(z_A,z_B) + \textstyle\sum_{m,n=1}^{N}\mathbf{G}_0(z_A,z_m)\cdot\mathbf{T}_{mn}^{(N)}\cdot\mathbf{G}_0(z_n,z_B)$$

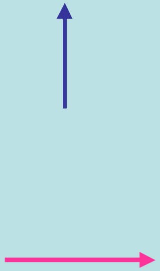


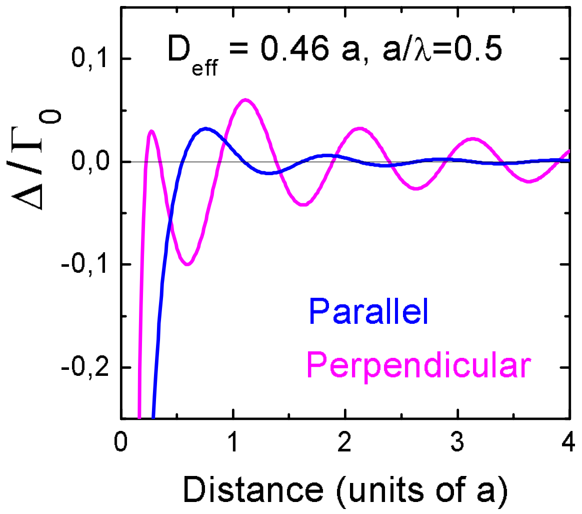
Wubs, Suttorp & Lagendijk, PRE **69**, 016616 (2004).



Emission rates and Lamb shifts near a plane







Measurements of position-dependent emission rates **in** and **near** dielectric:

in: Snoeks, Lagendijk & Polman (1995);

near: Ivanov, Cornelussen, Van Linden van den Heuvell & Spreeuw (2004)

Two-atom superradiance in free space

Theory:

Dicke, PR **93**, 99 (1954):

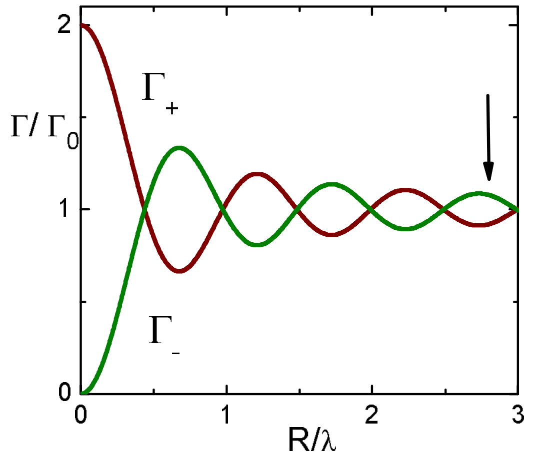
- Two decay rates, limiting values 0 and $2\Gamma0$.

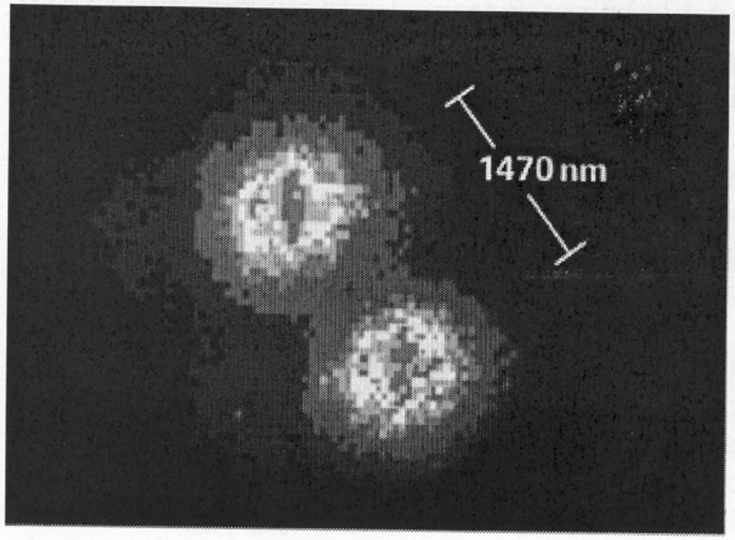
Experiment:

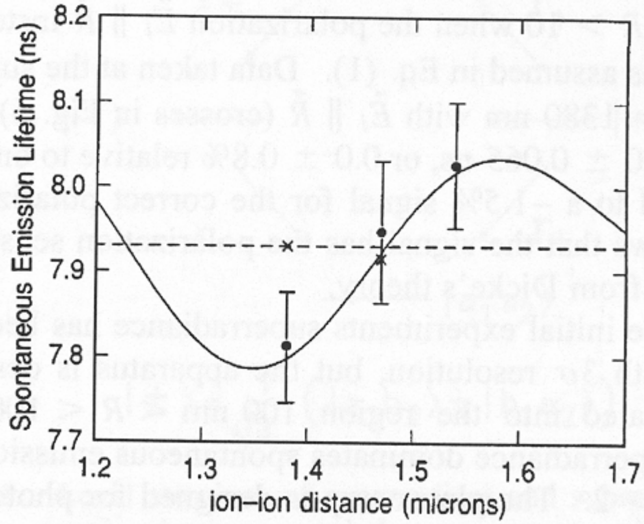
DeVoe & Brewer, PRL **76**, 2049 (1996):

A single pair of trapped atoms

λ=493 nm, R=1470 nm, lifetime change 1.5%



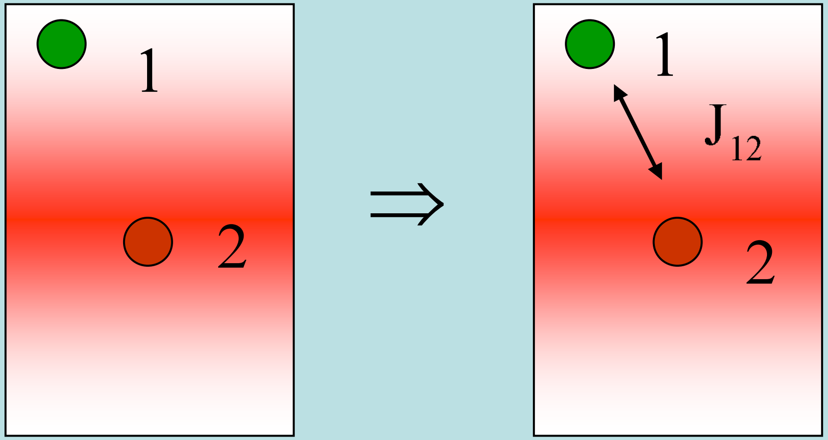




Two-atom superradiance in/near a dielectric

🟢 $\Omega_1 = \Omega + \Delta_1 - i\Gamma_1/2$

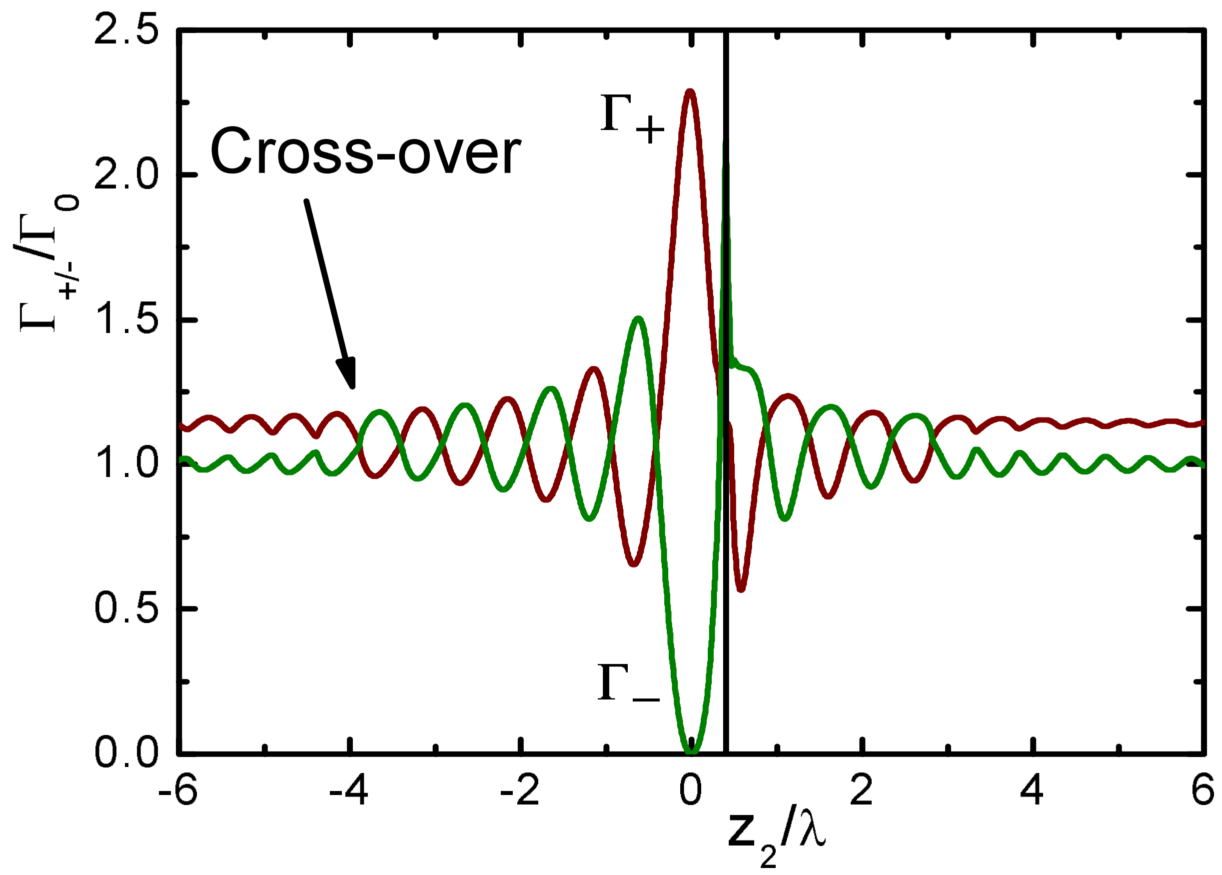
🔴 $\Omega_2 = \Omega + \Delta_2 - i\Gamma_2/2$

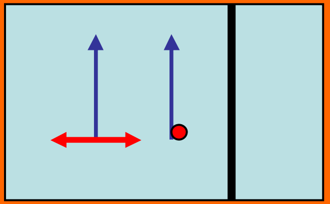


With interaction, again, two complex resonance frequencies:

$$\Omega_\pm = \Omega + \frac{\Delta_1+\Delta_2}{2} - i\,\frac{\Gamma_1+\Gamma_2}{4} \pm \sqrt{\underbrace{\left[\frac{\Delta_1-\Delta_2}{2} - i\,\frac{\Gamma_1-\Gamma_2}{4}\right]^2}_{\text{Medium-induced detuning}} + J_{12}^2}$$

Subradiance and superradiance near a surface





Wubs, Suttorp & Lagendijk, PRA **70**, 053823 (2004)

Part II:

QED in absorbing dielectric media

Extinction occurs in *all* dispersive media (Kramers & Kronig)

Commutation relations of quantum fields

$$\boldsymbol{E} = -\boldsymbol{\nabla}\Phi - \dot{\mathbf{A}}$$
$$\boldsymbol{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

Operator equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$$

Coulomb gauge

$$[\mathbf{A}(\mathbf{r}), \mathbf{\Pi}(\mathbf{r}')] = i\hbar\boldsymbol{\delta}_{\mathsf{T}}(\mathbf{r}-\mathbf{r}')$$

EM fields

$$[\mathbf{X}(\mathbf{r}), \mathbf{P}(\mathbf{r}')] = i\hbar\mathbf{I}\delta(\mathbf{r}-\mathbf{r}')$$

Polarization fields

$$\left[E_j(\mathbf{r}), B_k(\mathbf{r})\right] = \frac{i\hbar}{\varepsilon_0}\varepsilon_{jkl}\nabla'_l\delta(\mathbf{r}-\mathbf{r}')$$

Medium-independent
& nonlocal

Phenomenological theory: recipe (I/II)

1. Postulate a \`Langevin' equation:

$$-\vec{\nabla}\times\vec{\nabla}\times\boldsymbol{E}(\mathbf{r},\omega)+\varepsilon(\mathbf{r},\omega)\frac{\omega^2}{c^2}\boldsymbol{E}(\mathbf{r},\omega)=-i\mu_0\omega\boldsymbol{J}(\mathbf{r},\omega)$$

dissipation

fluctuation

Noise-current density

2. Solve for field operators:

$$\boldsymbol{E}(\boldsymbol{r},t)=-i\mu_0\int \mathrm{d}\mathbf{r}'\int_0^\infty \mathrm{d}\omega\, e^{-i\omega t}\omega\,\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\boldsymbol{J}(\mathbf{r}',\omega)+\text{h.c.}$$

Huttner & Barnett (1992); Scheel, Knoell & Welsch (1998);

Phenomenological theory: recipe (II/II)

3. *Assume* commutation relations:

$$\left[\boldsymbol{J}(\mathbf{r},\omega),\boldsymbol{J}^{\dagger}(\mathbf{r}',\omega')\right]=\frac{\hbar\varepsilon_0}{\pi}\,\mathrm{Im}[\varepsilon(\mathbf{r},\omega)]\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')\mathbf{I}$$

4. Check consistency with QED…

$$\left[E_j(\mathbf{r},t),B_k(\mathbf{r}',t)\right]=\frac{\hbar}{\pi\epsilon_0 c^2}\,\varepsilon_{klm}\nabla_l'\int_{-\infty}^{\infty}\mathrm{d}\omega\,\omega\,\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)$$

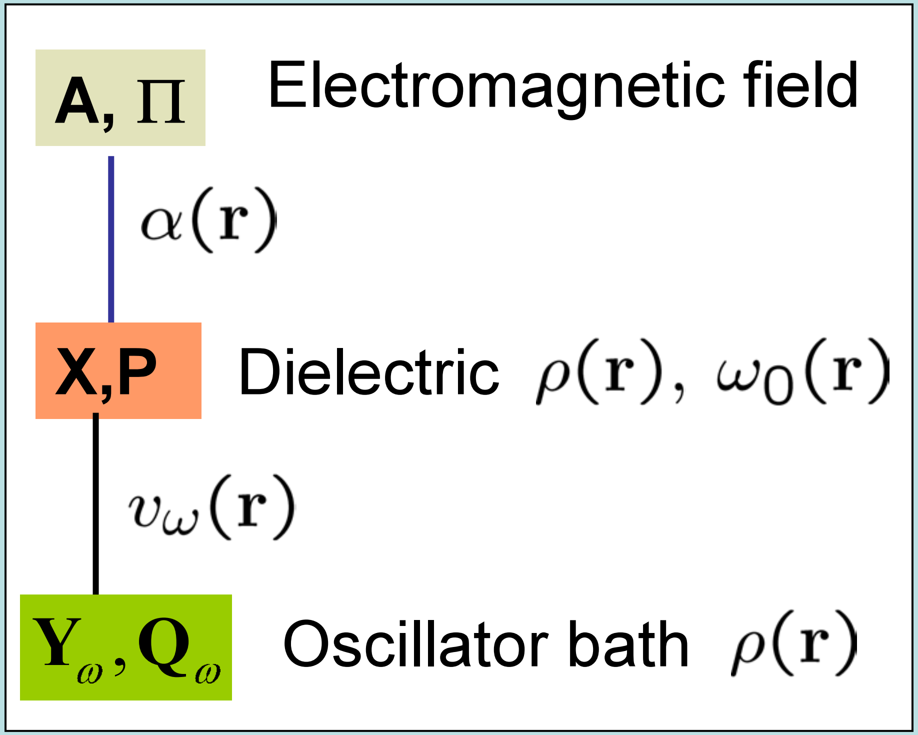
$$=\frac{i\hbar}{\varepsilon_0}\,\varepsilon_{jkl}\nabla_l'\delta(\mathbf{r}-\mathbf{r}')$$!

Huttner & Barnett (1992); Scheel, Knoell & Welsch (1998);

Microscopic theory

Goal: justify phenomenological theory

Model:



$\mathbf{A}, \Pi$ Electromagnetic field

$\alpha(\mathbf{r})$

$\mathbf{X}, \mathbf{P}$ Dielectric $\rho(\mathbf{r}),\ \omega_0(\mathbf{r})$

$v_\omega(\mathbf{r})$

$\mathbf{Y}_\omega, \mathbf{Q}_\omega$ Oscillator bath $\rho(\mathbf{r})$

Methods:

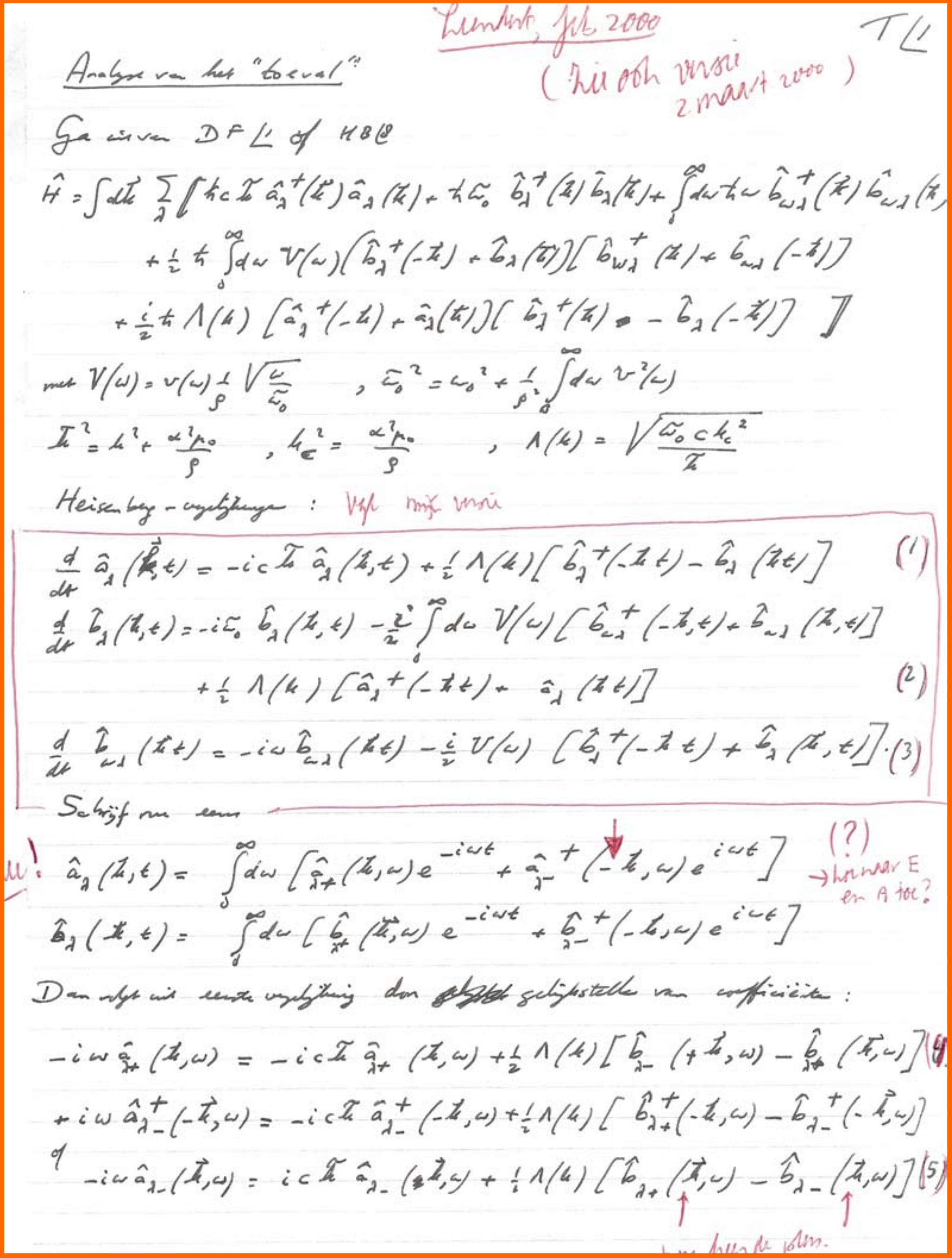
Fano diagonalization (1,5),
path integrals (2), or
Laplace transformations (3,4)

Translation-invariant dielectric:

1. Huttner & Barnett (1992);
2. Bechler (1999)
3. Wubs & Suttorp (2001).

Inhomogeneous dielectric:

4. Suttorp & Wubs (2004)
5. Suttorp & Van Wonderen (2004)



Lunteren, feb 2000

(Zie ook versie 2 maart 2000)

TL1

Analyse van het "toeval"

Ga uit van DFL of HBE

$$\hat{H} = \int d\vec{k} \sum_\lambda \Big[\hbar c \tilde{k}\, \hat{a}_\lambda^+(\vec{k})\hat{a}_\lambda(\vec{k}) + \hbar\tilde{\omega}_0\, \hat{b}_\lambda^+(\vec{k})\hat{b}_\lambda(\vec{k}) + \int_0^\infty d\omega\, \hbar\omega\, \hat{b}_{\omega\lambda}^+(\vec{k})\hat{b}_{\omega\lambda}(\vec{k})$$

$$+ \tfrac{1}{2}\hbar \int_0^\infty d\omega\, V(\omega)\big(\hat{b}_\lambda^+(-\vec{k}) + \hat{b}_\lambda(\vec{k})\big)\big[\hat{b}_{\omega\lambda}^+(\vec{k}) + \hat{b}_{\omega\lambda}(-\vec{k})\big]$$

$$+ \tfrac{i}{2}\hbar\, \Lambda(k)\, \big[\hat{a}_\lambda^+(-\vec{k}) + \hat{a}_\lambda(\vec{k})\big]\big[\hat{b}_\lambda^+(\vec{k}) - \hat{b}_\lambda(-\vec{k})\big] \Big]$$

met $V(\omega) = v(\omega)\frac{1}{\rho}\sqrt{\frac{\omega}{\tilde{\omega}_0}}$, $\tilde{\omega}_0^2 = \omega_0^2 + \frac{1}{\rho^2}\int_0^\infty d\omega\, v^2(\omega)$

$\tilde{k}^2 = k^2 + \frac{\alpha^2\mu_0}{\rho}$, $k_c^2 = \frac{\alpha^2\mu_0}{\rho}$, $\Lambda(k) = \sqrt{\frac{\tilde{\omega}_0 c k_c^2}{\tilde{k}}}$

Heisenberg-vergelijkingen : Vgl. mijn versie

$$\frac{d}{dt}\hat{a}_\lambda(\vec{k},t) = -ic\tilde{k}\,\hat{a}_\lambda(\vec{k},t) + \tfrac{1}{2}\Lambda(k)\big[\hat{b}_\lambda^+(-\vec{k}t) - \hat{b}_\lambda(\vec{k}t)\big] \quad (1)$$

$$\frac{d}{dt}\hat{b}_\lambda(\vec{k},t) = -i\tilde{\omega}_0\,\hat{b}_\lambda(\vec{k},t) - \frac{i}{2}\int_0^\infty d\omega\, V(\omega)\big[\hat{b}_{\omega\lambda}^+(-\vec{k},t) + \hat{b}_{\omega\lambda}(\vec{k},t)\big]$$

$$+ \tfrac{1}{2}\Lambda(k)\big[\hat{a}_\lambda^+(-\vec{k}t) + \hat{a}_\lambda(\vec{k}t)\big] \quad (2)$$

$$\frac{d}{dt}\hat{b}_{\omega\lambda}(\vec{k}t) = -i\omega\,\hat{b}_{\omega\lambda}(\vec{k}t) - \frac{i}{2}V(\omega)\big[\hat{b}_\lambda^+(-\vec{k}t) + \hat{b}_\lambda(\vec{k},t)\big]. \quad (3)$$

Schrijf nu eens

M! $\hat{a}_\lambda(\vec{k},t) = \int_0^\infty d\omega\,\big[\hat{a}_{\lambda+}(\vec{k},\omega)e^{-i\omega t} + \hat{a}_{\lambda-}^+(\overset{\downarrow}{-}\vec{k},\omega)e^{i\omega t}\big]$ (?) → hoe naar E en A toe?

$\hat{b}_\lambda(\vec{k},t) = \int_0^\infty d\omega\,\big[\hat{b}_{\lambda+}(\vec{k},\omega)e^{-i\omega t} + \hat{b}_{\lambda-}^+(-\vec{k},\omega)e^{i\omega t}\big]$

Dan volgt uit eerste vergelijking door ~~[illegible]~~ gelijkstellen van coefficiënten :

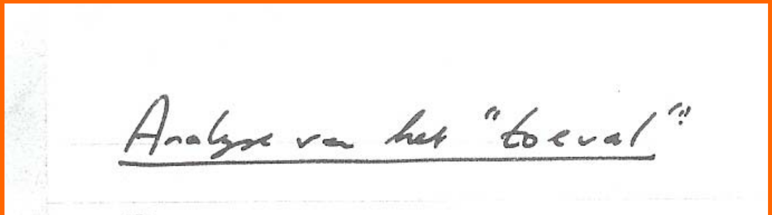
$$-i\omega\,\hat{a}_{\lambda+}(\vec{k},\omega) = -ic\tilde{k}\,\hat{a}_{\lambda+}(\vec{k},\omega) + \tfrac{1}{2}\Lambda(k)\big[\hat{b}_{\lambda-}(+\vec{k},\omega) - \hat{b}_{\lambda+}(\vec{k},\omega)\big] \quad (4)$$

$$+i\omega\,\hat{a}_{\lambda-}^+(-\vec{k},\omega) = -ic\tilde{k}\,\hat{a}_{\lambda-}^+(-\vec{k},\omega) + \tfrac{1}{2}\Lambda(k)\big[\hat{b}_{\lambda+}^+(-\vec{k},\omega) - \hat{b}_{\lambda-}^+(-\vec{k},\omega)\big]$$

of

$$-i\omega\,\hat{a}_{\lambda-}(\vec{k},\omega) = ic\tilde{k}\,\hat{a}_{\lambda-}(\vec{k},\omega) + \tfrac{1}{2}\Lambda(k)\big[\hat{b}_{\lambda+}(\underset{\uparrow}{\vec{k}},\omega) - \hat{b}_{\lambda-}(\underset{\uparrow}{\vec{k}},\omega)\big] \quad (5)$$

beide plus.

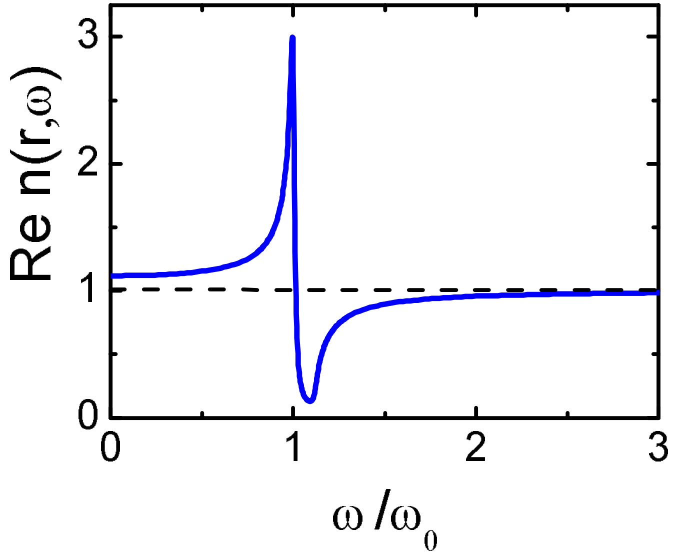


Analyse van het "toeval"

Microscopic theory: results (I/III)

$$\varepsilon(\mathbf{r},\omega) = n^2(\mathbf{r},\omega) = 1 + \chi(\mathbf{r},\omega)$$

in terms of model parameters



Noise-current density:

$$\bar{\boldsymbol{J}}(p) = -\frac{1}{\mu_0 p}\nabla\times[\nabla\times\boldsymbol{A}(0)]+\boldsymbol{\Pi}(0)+\alpha\left[1-\frac{\varepsilon_0\rho}{\alpha^2}\bar{\chi}(p)\right]\mathbf{X}(0)- [\alpha\mathbf{X}(0)]_{\mathrm{L}}-\frac{\varepsilon_0}{\alpha}p\bar{\chi}(p)\boldsymbol{P}(0)-\frac{\varepsilon_0}{\alpha}p\bar{\chi}(p)\int_0^\infty \mathrm{d}\omega\,\frac{v_\omega}{p^2+\omega^2}\left[\omega^2\boldsymbol{Y}(0)-\frac{p}{\rho}\boldsymbol{Q}_\omega(0)\right]$$

Microscopic theory: results (II/III)

$$2\pi \boldsymbol{E}(\omega) \quad = \quad \bar{\boldsymbol{E}}(-i\omega+0) \quad + \quad \breve{\boldsymbol{E}}(i\omega+0)$$

Fourier = positive-time + negative-time Laplace

$$\boxed{-\vec{\nabla}\times\vec{\nabla}\times\bar{\boldsymbol{E}}(\mathbf{r},-i\omega+0)+\varepsilon(\mathbf{r},\omega)\frac{\omega^2}{c^2}\bar{\boldsymbol{E}}(\mathbf{r},-i\omega+0)=-i\mu_0\omega\bar{\boldsymbol{J}}(\mathbf{r},-i\omega+0)}$$

$$\boxed{-\vec{\nabla}\times\vec{\nabla}\times\breve{\boldsymbol{E}}(\mathbf{r},i\omega+0)+\varepsilon^*(\mathbf{r},\omega)\frac{\omega^2}{c^2}\breve{\boldsymbol{E}}(\mathbf{r},i\omega+0)=-i\mu_0\omega\breve{\boldsymbol{J}}(\mathbf{r},i\omega+0)}$$

+

$$-\vec{\nabla}\times\vec{\nabla}\times\boldsymbol{E}(\mathbf{r},\omega)+\varepsilon(\mathbf{r},\omega)\frac{\omega^2}{c^2}\boldsymbol{E}(\mathbf{r},\omega)\equiv-i\mu_0\omega\boldsymbol{J}(\mathbf{r},\omega)$$

But: $2\pi\boldsymbol{J}(\omega)\neq\bar{\boldsymbol{J}}(-i\omega+0)+\breve{\boldsymbol{J}}(i\omega+0)$!

Microscopic theory: results (III/III)

Local commutation relation derived:

$$\left[\boldsymbol{J}(\mathbf{r},\omega),\boldsymbol{J}^{\dagger}(\mathbf{r}',\omega')\right]=\frac{\hbar\varepsilon_0}{\pi}\,\mathrm{Im}[\varepsilon(\mathbf{r},\omega)]\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')\mathbf{I}$$

Simple form of Hamiltonian:

$$H=\frac{\pi}{\varepsilon_0}\int \mathrm{d}\mathbf{r}\int_0^{\infty}\mathrm{d}\omega\,\frac{\boldsymbol{J}^{\dagger}(\mathbf{r},\omega)\cdot\boldsymbol{J}(\mathbf{r},\omega)}{\omega\,\mathrm{Im}[\varepsilon(\mathbf{r},\omega)]}$$

=> Justification of phenomenological theory

Conclusions

- Photonic medium strongly modifies 1-atom *and* superradiant emission rates
- Phenomeological QED for inhomogeneous absorbing media is justified by a microscopic model

Thanks for your attention!

Last but not least:

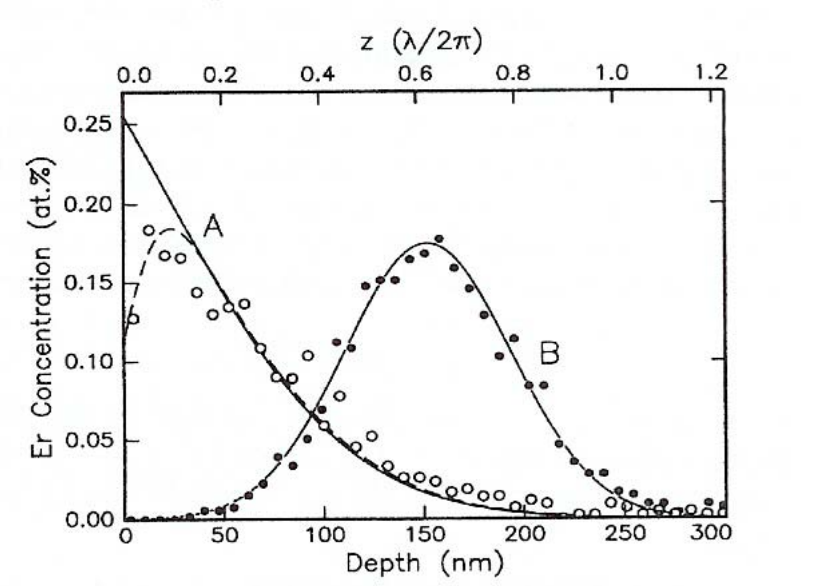
Beste Leendert,

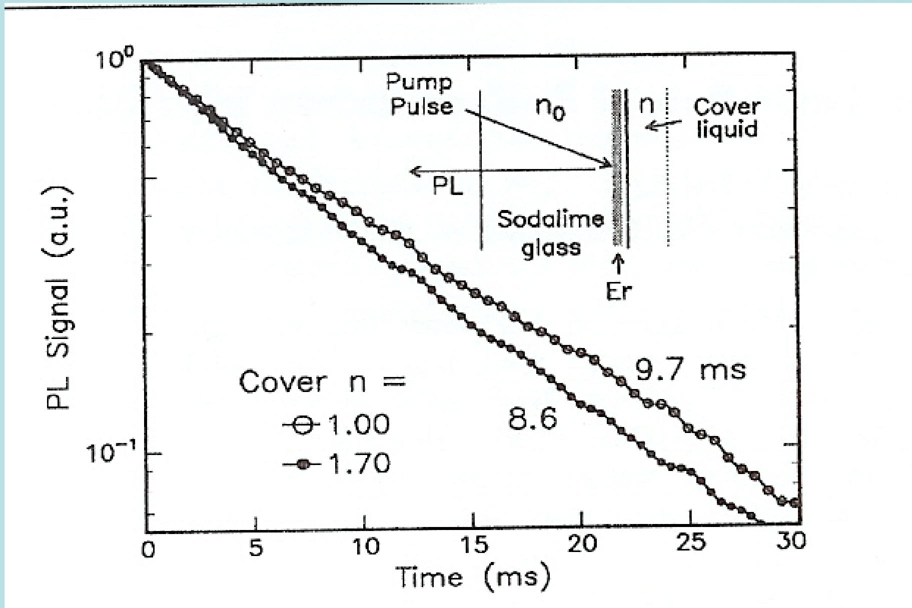
Dank voor de
goede begeleiding

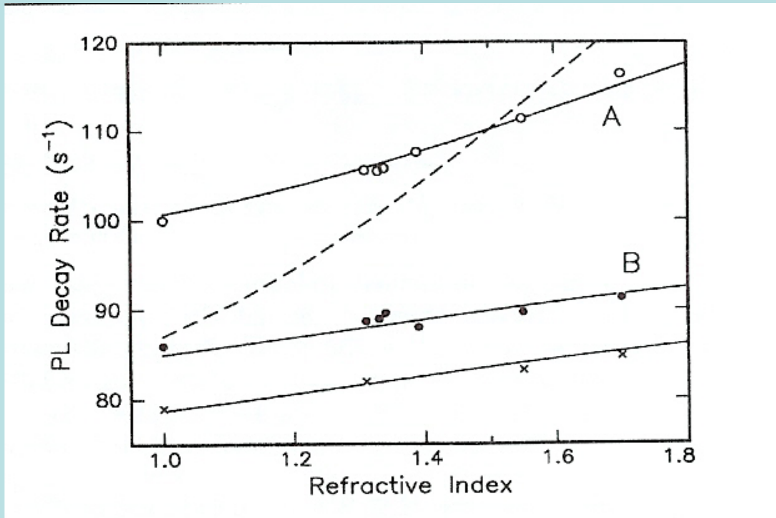
en voor de
aangename samenwerking,

en van harte gefeliciteerd
met je 65^e verjaardag!

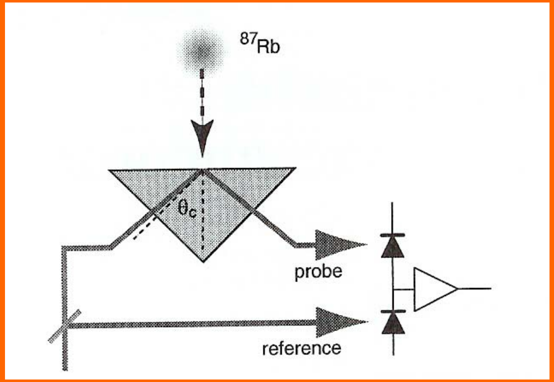
Figures from Snoeks et al, PRL **74**, 2459 (1995)

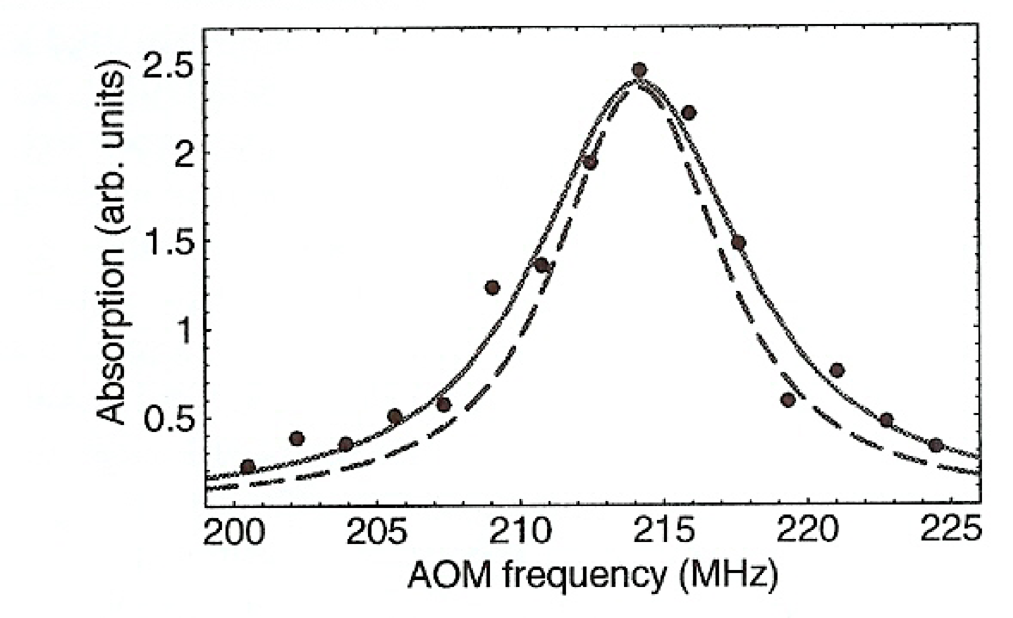




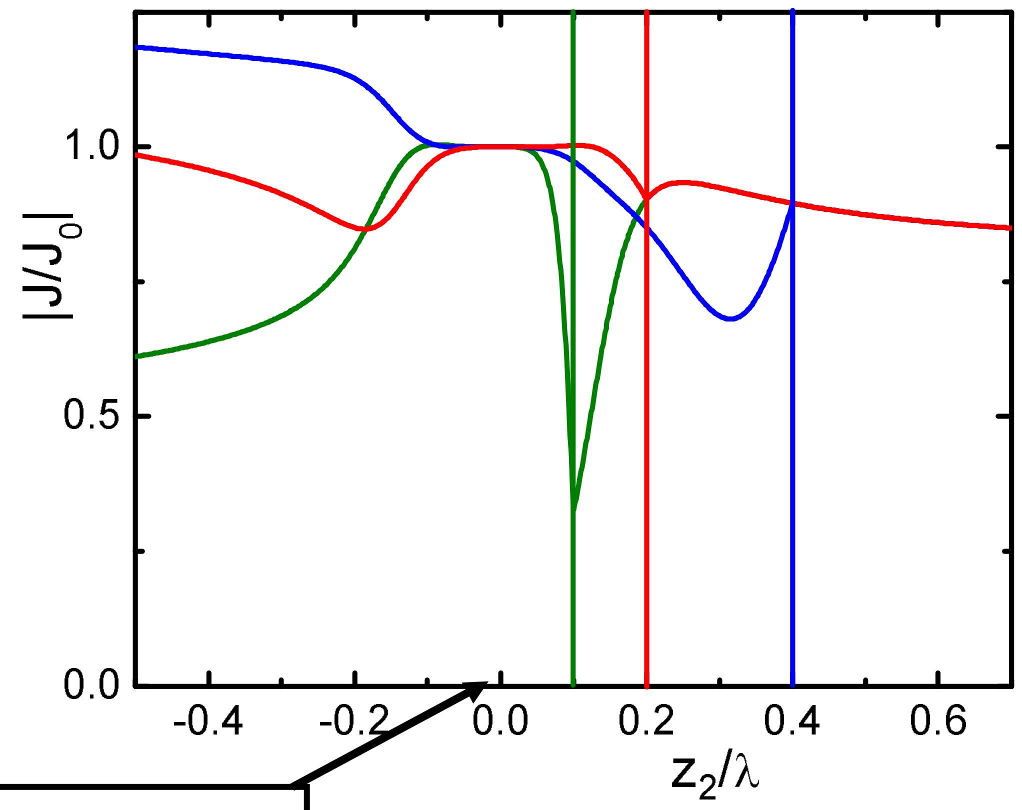


Figures from Ivanov et al, J. Opt. B: Quantum Semiclass.Opt. **6**, 454 (2004)





Modified dipole-dipole interaction

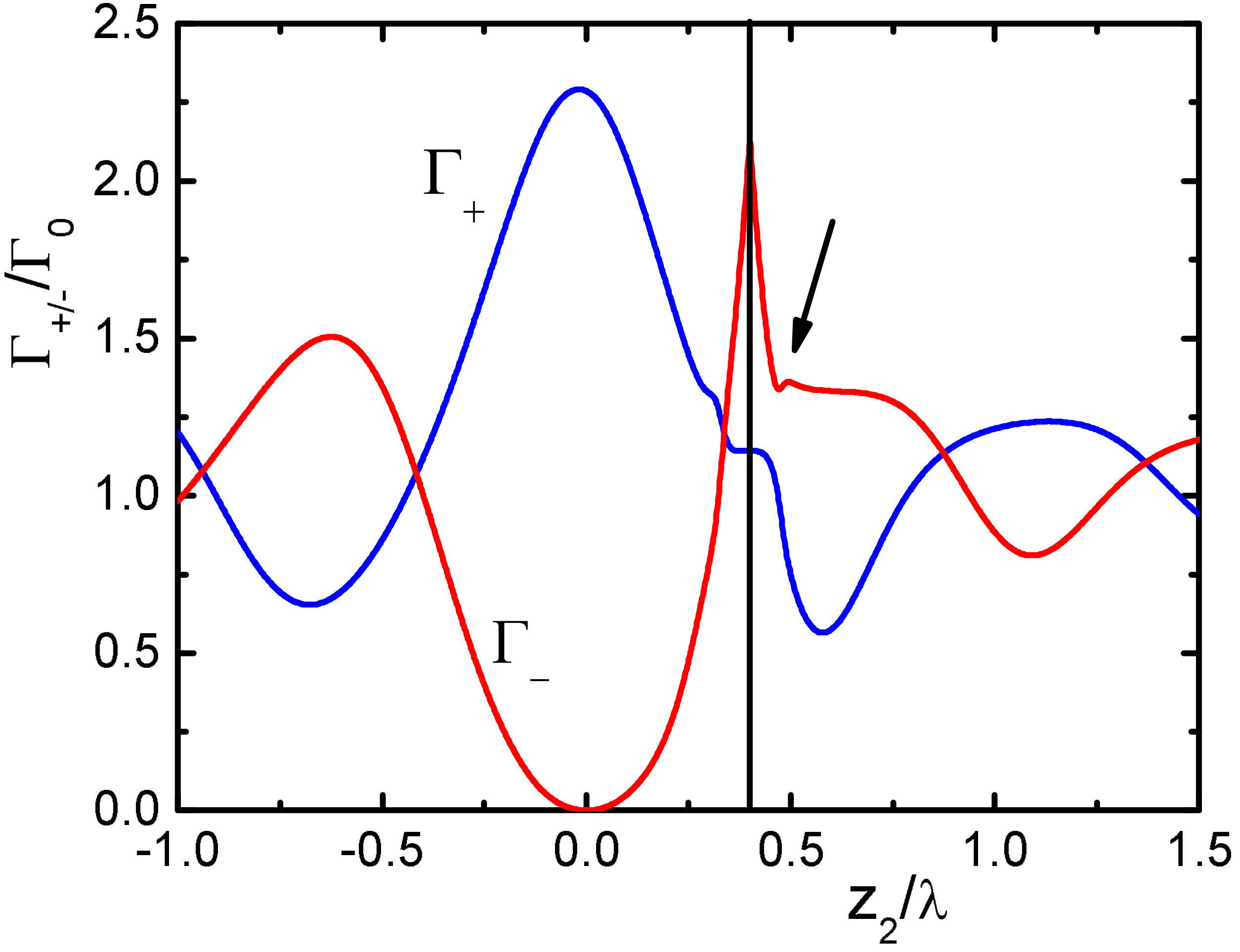


Planes fixed

Atom 1 fixed

Atom 2 moves

Subradiance and superradiance (I)



Lagrangian density

$$\boxed{\mathcal{L} = \mathcal{L}_{\mathsf{light}} + \mathcal{L}_{\mathsf{matter}} + \mathcal{L}_{\mathsf{bath}} + \mathcal{L}_{\mathsf{l-m}} + \mathcal{L}_{\mathsf{m-b}}}$$

$$\mathcal{L}_{\mathsf{light}}(\mathbf{r}) = \tfrac{1}{2}\varepsilon_0 E^2(\mathbf{r}) - \tfrac{1}{2}\mu_0^{-1} B^2(\mathbf{r})$$

$$\mathcal{L}_{\mathsf{matter}}(\mathbf{r}) = \tfrac{1}{2}\rho(\mathbf{r})\dot{X}^2(\mathbf{r}) - \tfrac{1}{2}\rho(\mathbf{r})\omega_0^2(\mathbf{r})X^2(\mathbf{r})$$

$$\mathcal{L}_{\mathsf{bath}}(\mathbf{r}) = \tfrac{1}{2}\rho(\mathbf{r}) \textstyle\int_0^\infty \mathsf{d}\omega \left[\dot{Y}_\omega^2(\mathbf{r}) - \omega^2 Y_\omega^2(\mathbf{r})\right]$$

$$\mathcal{L}_{\mathsf{l-m}}(\mathbf{r}) = -\Phi(\mathbf{r})\nabla\cdot[\alpha(\mathbf{r})\boldsymbol{X}(\mathbf{r})] - \alpha(\mathbf{r})\boldsymbol{A}(\mathbf{r})\cdot\dot{\boldsymbol{X}}(\mathbf{r})$$

$$\mathcal{L}_{\mathsf{m-b}}(\mathbf{r}) = -\textstyle\int_0^\infty \mathsf{d}\omega\, v_\omega(\mathbf{r})\boldsymbol{X}(\mathbf{r})\cdot\dot{\boldsymbol{Y}}_\omega(\mathbf{r})$$