Landau-Zener transitions in circuit QED

Martijn Wubs

INSTITUT für PHYSIK
UNIVERSITÄT AUGSBURG

QIP 2006, Herrsching

Many thanks to:

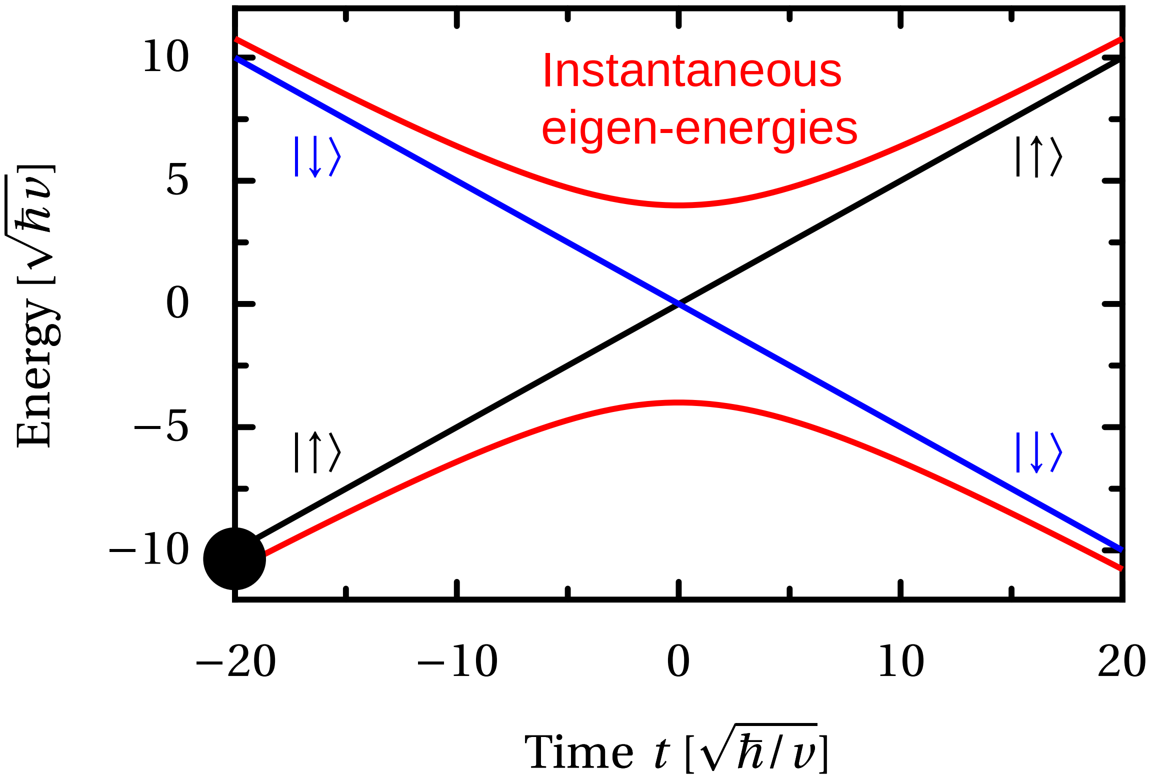
Keiji Saito	(Tokyo)
Sigmund Kohler	(Augsburg)
Yosuke Kayanuma	(Osaka)
Peter Hänggi	(Augsburg)

Overview

- What is a Landau-Zener transition?
- LZ transitions in circuit QED
- Effects of dissipation

Landau-Zener transition

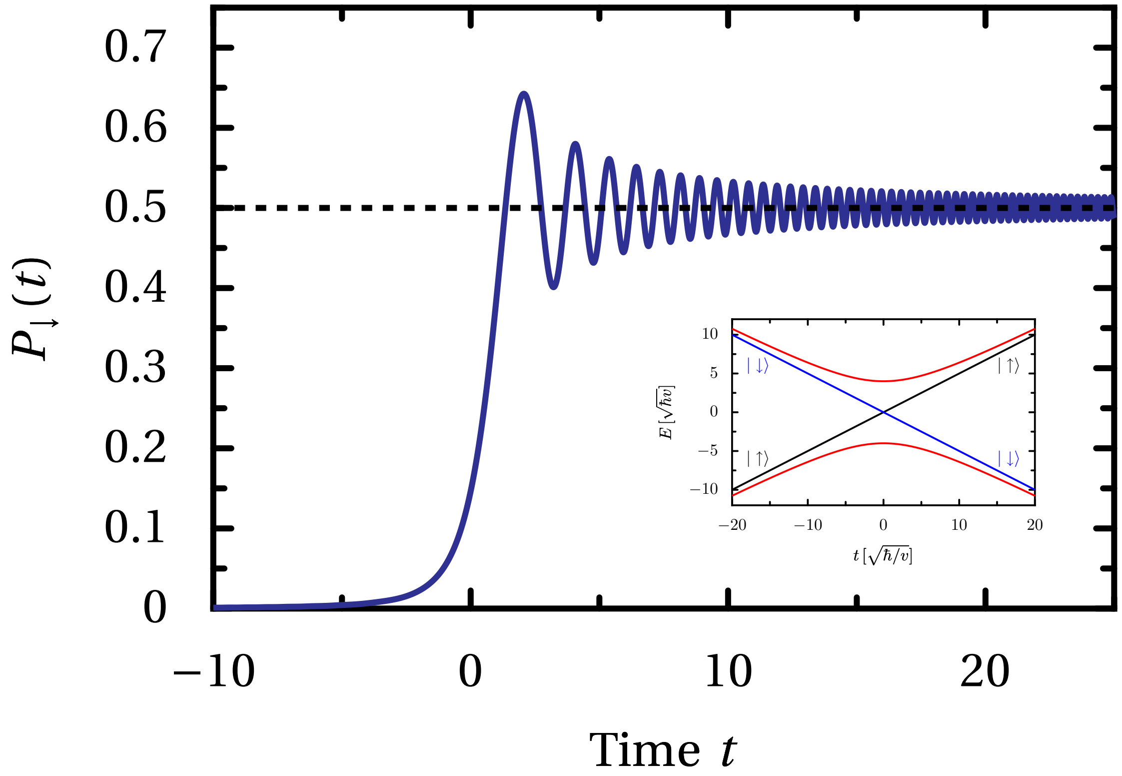
Driven two-level system: $H(t) = \frac{vt}{2}\,\boldsymbol{\sigma}_z + g\,\boldsymbol{\sigma}_x$



Gap = 2 g

Initial state ⟶ Final state ?

Landau-Zener transition



$$P_\downarrow(\infty)$$

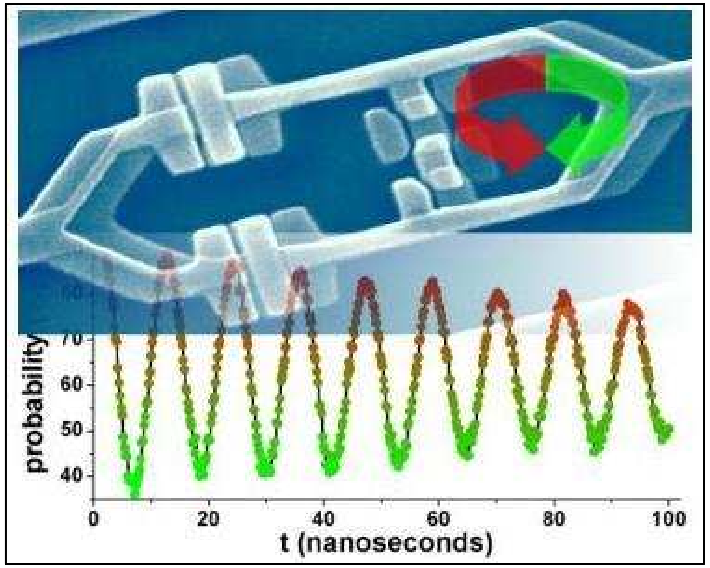
$$= 1 - \exp\left(-\frac{2\pi g^2}{\hbar v}\right)$$

This example: $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\chi} \end{pmatrix}$

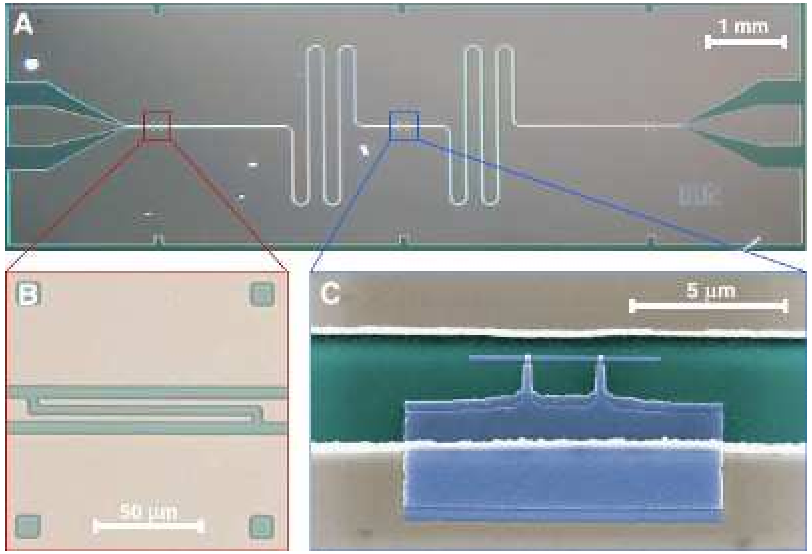
Landau (1932); Zener (1932); Stueckelberg (1932).

Circuit cavity QED: superconducting qubit coupled to a harmonic oscillator

1.



2.



Flux qubit coupled to SQUID

I. Chiorescu *et al.*, Nature **431**, 159 (2004).

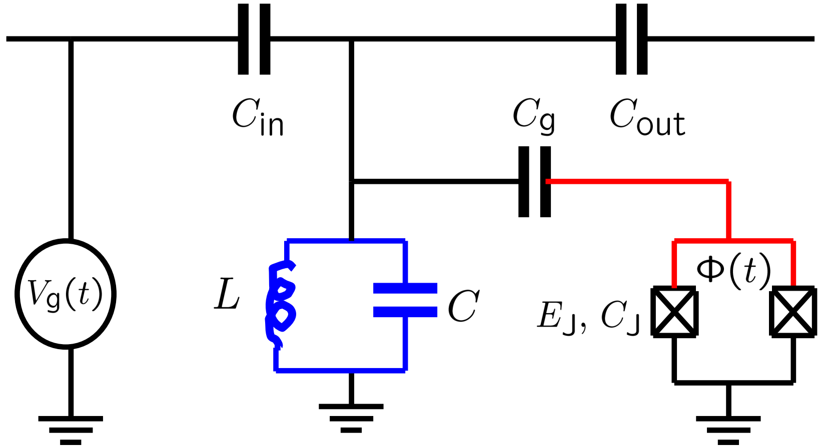
Cooper-pair box coupled to 1D-resonator

A. Wallraff *et al.*, Nature **431**, 162 (2004).

LZ transitions in superconducting qubits

Izmalkov *et al.* EPL (2004); Oliver *et al.,* Science (2005); Sillanpää *et* al., cond-mat/0510559

The tunable Hamiltonian of circuit cavity QED



Resonator = Oscillator

Cooper-pair box = Qubit

Blais *et al.*, PRA (2004):

$$H_{\mathrm{Q}}(t) = -\frac{E_{\mathrm{J}}(t)}{2}\boldsymbol{\sigma}_z - \frac{E_{\mathrm{el}}(t)}{2}\boldsymbol{\sigma}_x$$

$$H_{\mathrm{O}} = \hbar\Omega\, b^\dagger b$$

$$H_{\mathrm{QO}}(t) = \gamma\,[\boldsymbol{\sigma}_x - 1 + 2N_{\mathrm{g}}(t)]\,(b + b^\dagger)$$

Parameters:

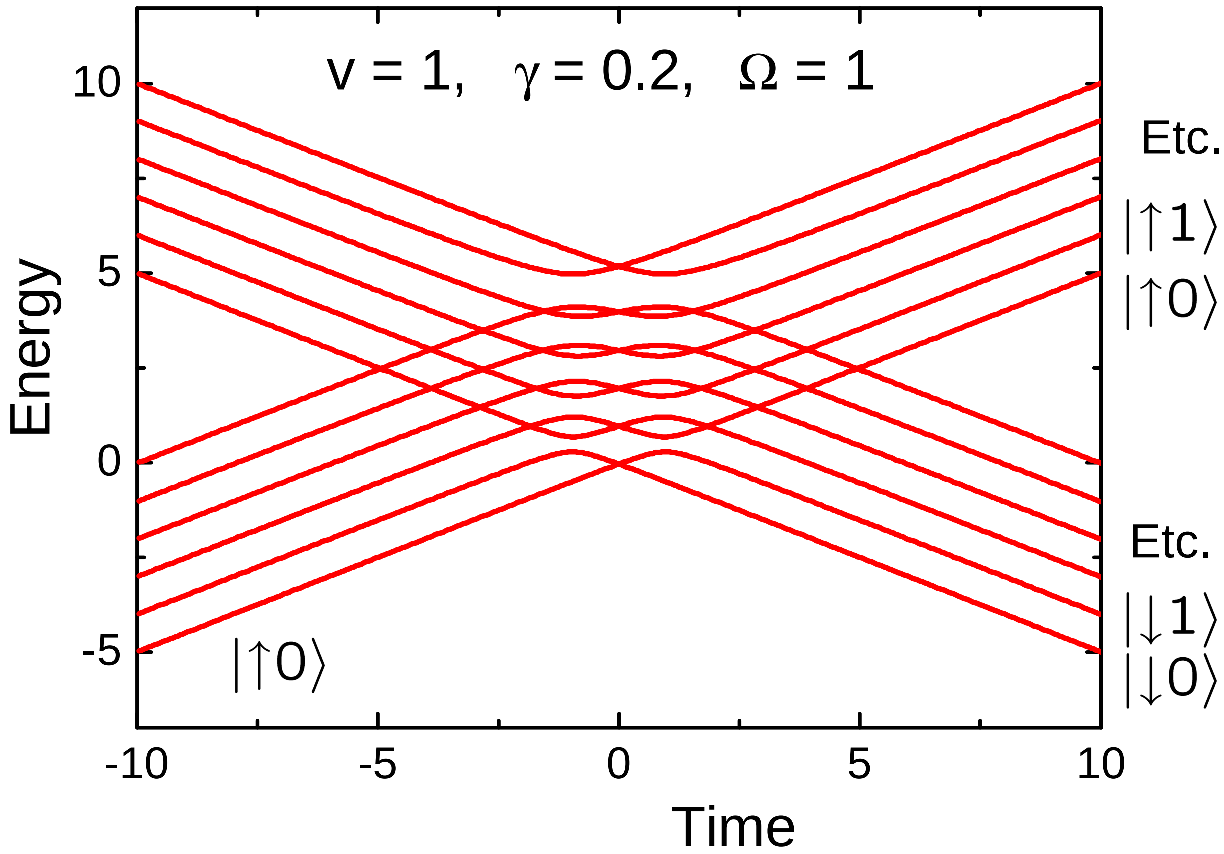
$$E_{\mathrm{el}}(t) = 4E_{\mathrm{C}}\,[1 - 2N_{\mathrm{g}}(t)] = 0$$

$$E_{\mathrm{J}}(t) = E_{\mathrm{J}}\cos\left[\pi\Phi(t)/\Phi_0\right]/2 \overset{!}{=} -vt$$

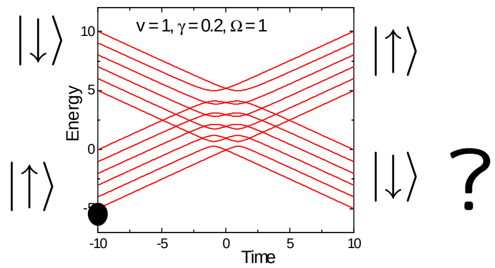
$$\Rightarrow \quad H(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + \gamma\,\boldsymbol{\sigma}_x(b + b^\dagger) + \hbar\Omega\, b^\dagger b$$

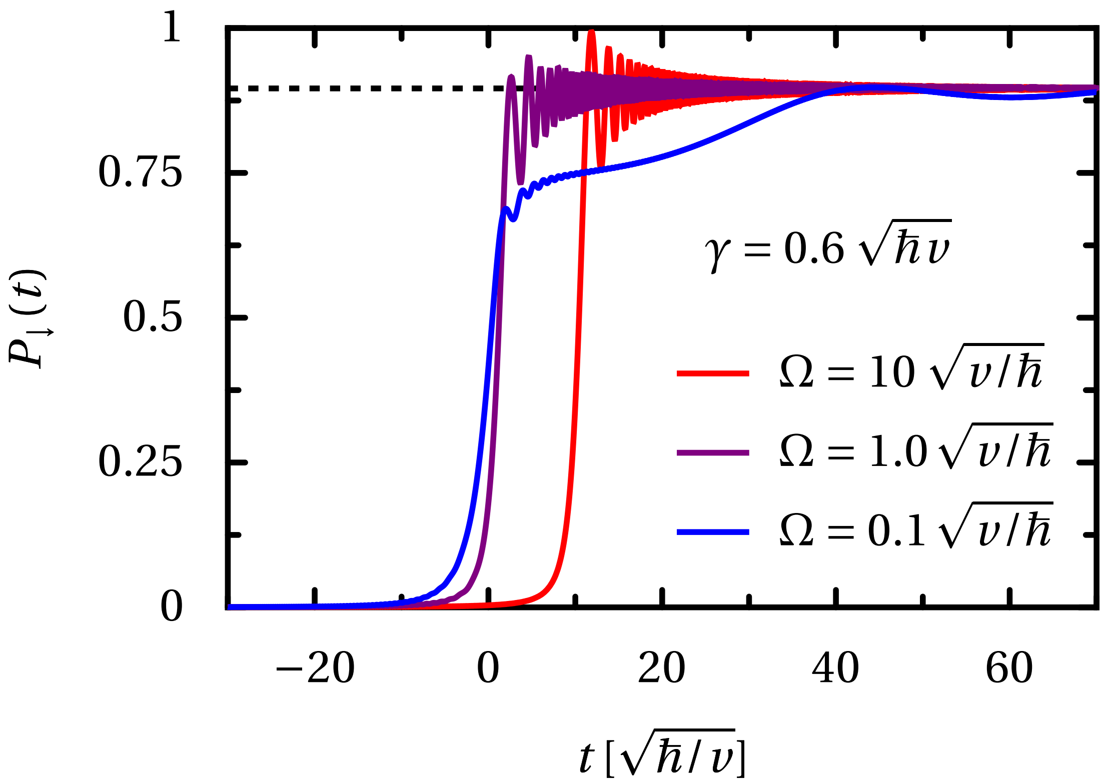
Instantaneous energies

$$H(t) = \frac{vt}{2}\,\boldsymbol{\sigma}_z + \gamma\,\boldsymbol{\sigma}_x(b + b^\dagger) + \hbar\Omega\, b^\dagger b$$



Dynamics when oscillator starts in ground state

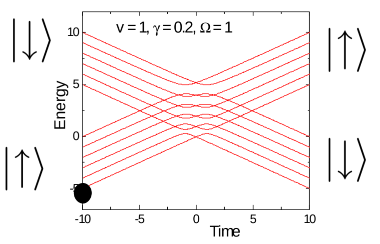




$$P_\downarrow(\infty) = 1 - \exp\left[-\frac{2\pi\gamma^2}{\hbar v}\right]$$

Exact result,
Beyond RWA,
NO
frequency
dependence!

Calculation of transition probabilities

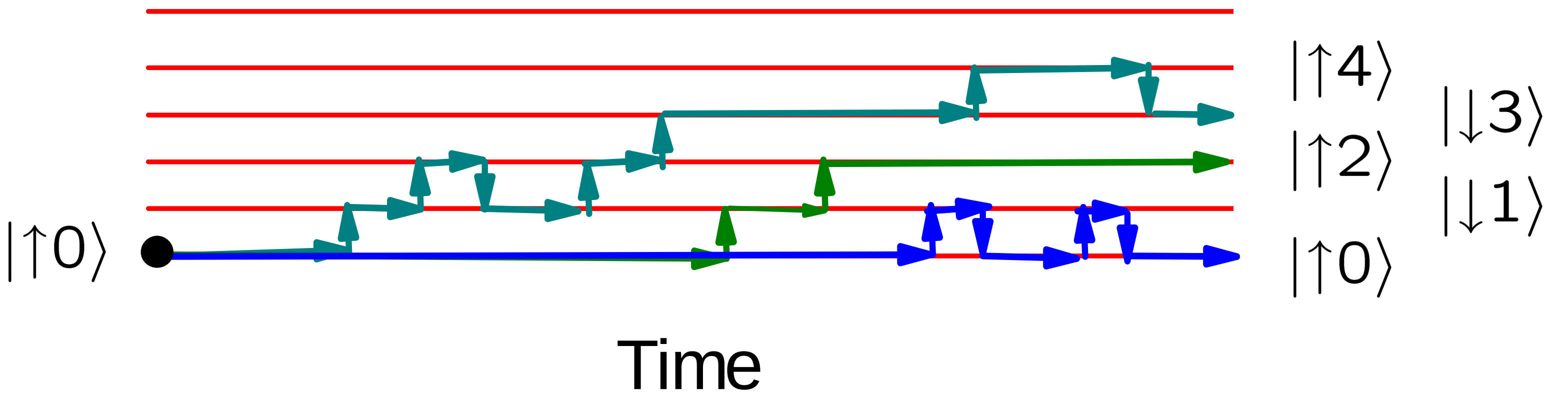


?

$$|\tilde{\psi}(\infty)\rangle = \overleftarrow{T} \exp\left[-\frac{i}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t\, \tilde{V}(t)\right] |\uparrow 0\rangle$$

$$\tilde{V}(t) = \gamma\, \sigma_x (b^\dagger e^{i\Omega t} + b\, e^{-i\Omega t}) \exp(-ivt^2\sigma_z/2\hbar)$$

Contribution? No Yes No

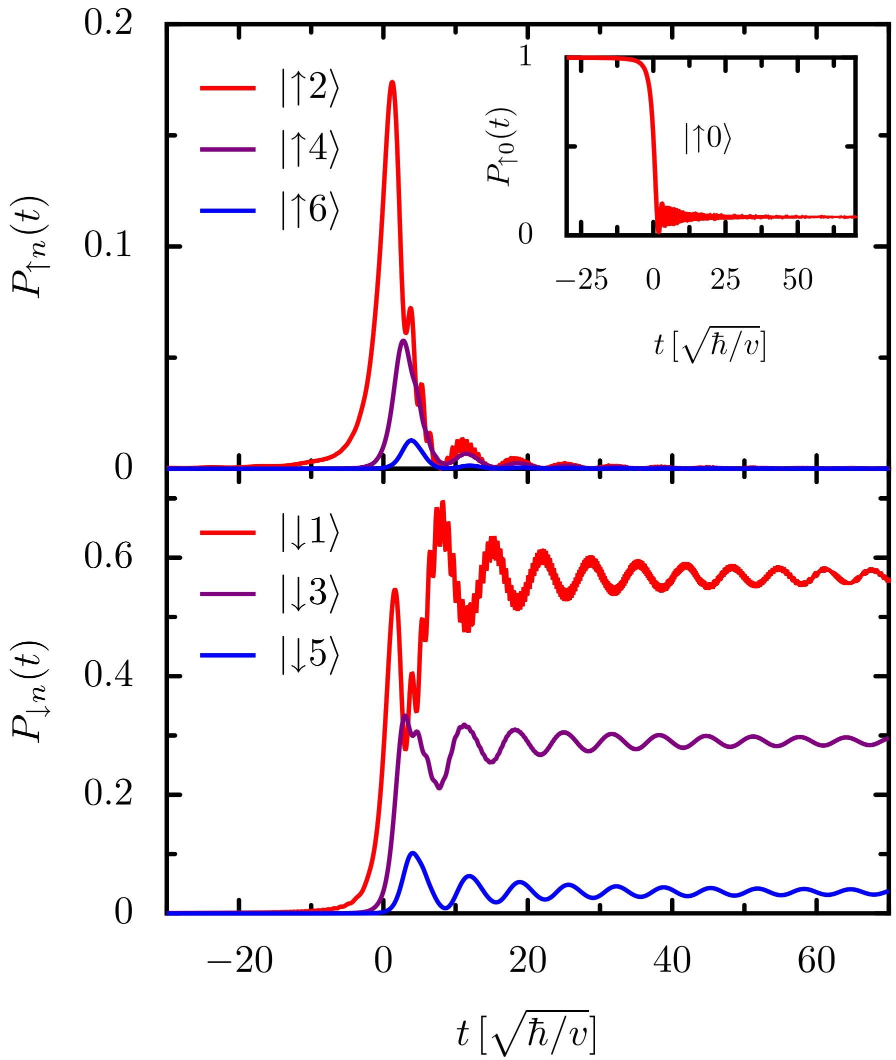


Selection rule:

Only (repeated) up-down processes contribute to $P_\uparrow(\infty)$

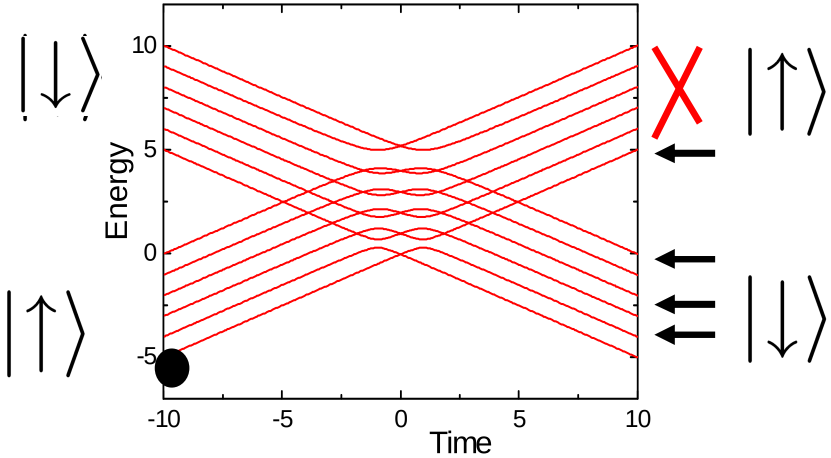
Saito, Wubs, Kohler, Kayanuma, & Hänggi, cond-mat/0603188

Qubit-oscillator entanglement



$\gamma = 0.6\,\sqrt{\hbar v}$

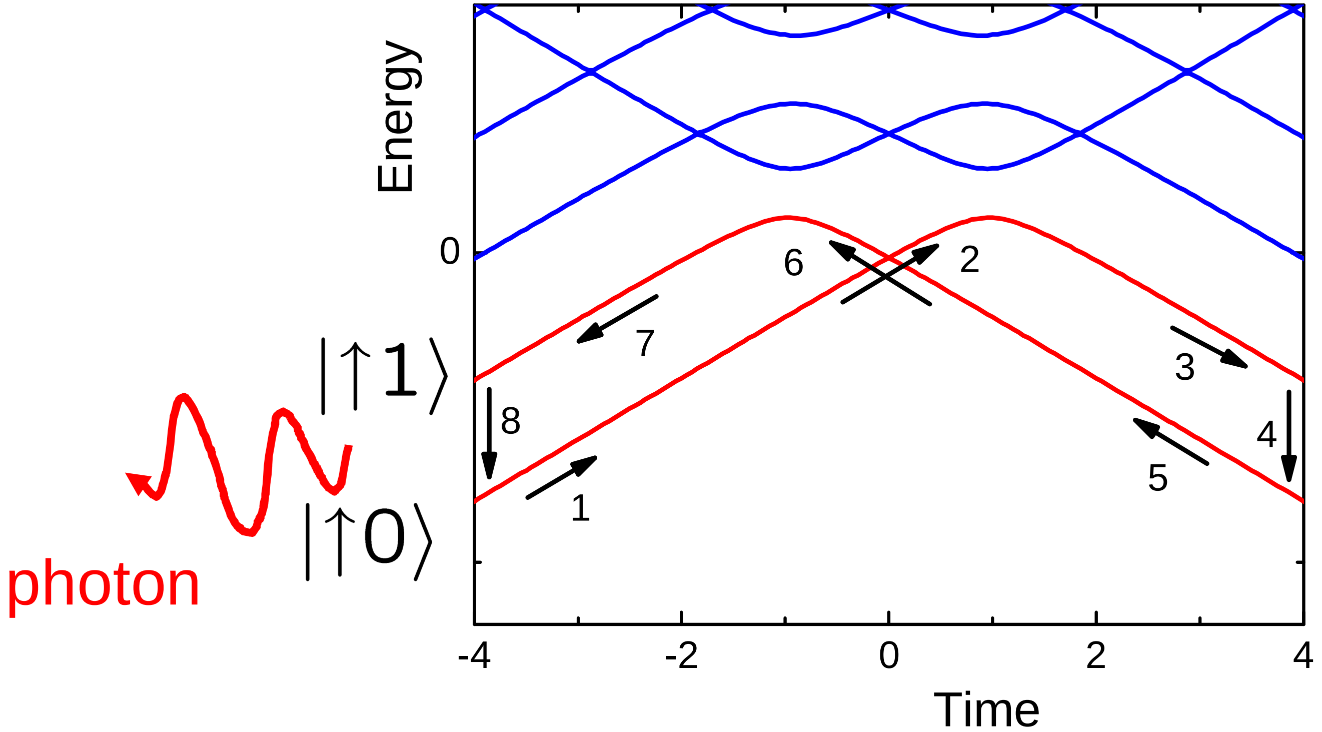
$\Omega = 0.5\,\sqrt{v/\hbar}$



$$|\psi(\infty)\rangle = \sqrt{1 - P_{\mathrm{LZ}}}|\uparrow 0\rangle + \sqrt{P_{\mathrm{LZ}}}\,(c_1|\downarrow 1\rangle + c_3|\downarrow 3\rangle + \ldots)$$

✗ : “No-go theorem”
Volkov & Ostrovsky ‘05

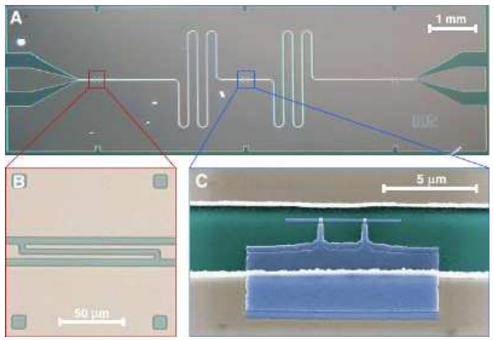
LZ cycle for single-photon generation



Steps 1-3:

$E_\mathrm{J}(t) = -vt$

- Effective 4-level problem $\hbar\Omega \gg k_\mathrm{B}T$
- Adiabatic limit $\gamma^2/\hbar v \gg 1$



Semiclassical intermezzo

$$H_{\mathsf{Q+O}}(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + \hbar\Omega\, b^\dagger b + \gamma\, \boldsymbol{\sigma}_x (b + b^\dagger)$$

Assume oscillator in coherent state $|\alpha(t)\rangle$ unaffected by qubit:

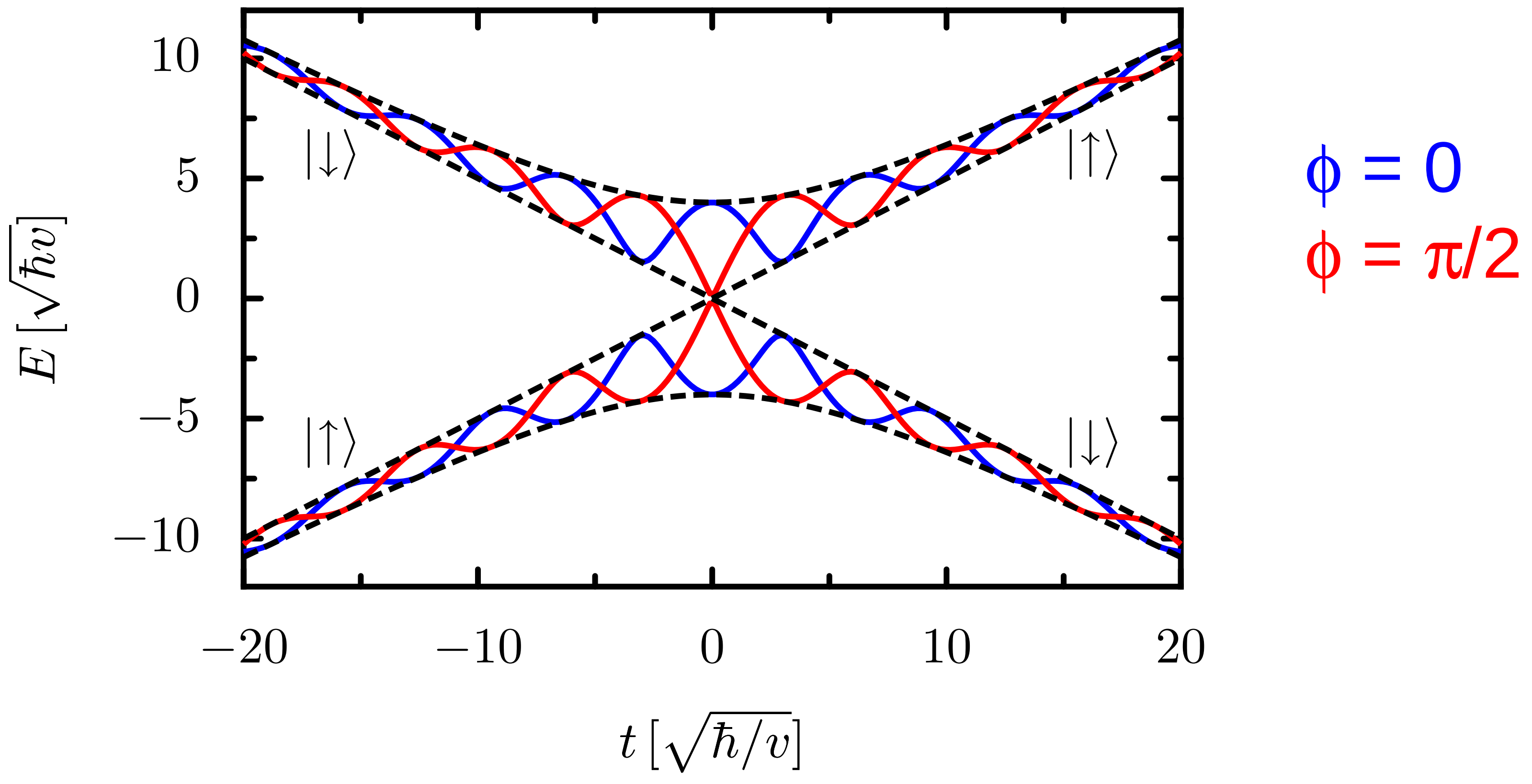
$$\alpha(t) = |\alpha| e^{-i\Omega t - i\phi}$$

$$\Rightarrow \quad H_{\mathsf{Q}}(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + g \cos(\Omega t + \phi)\boldsymbol{\sigma}_x$$

with $\quad g = 2|\alpha|\gamma$

"Landau-Zener meets Rabi problem"

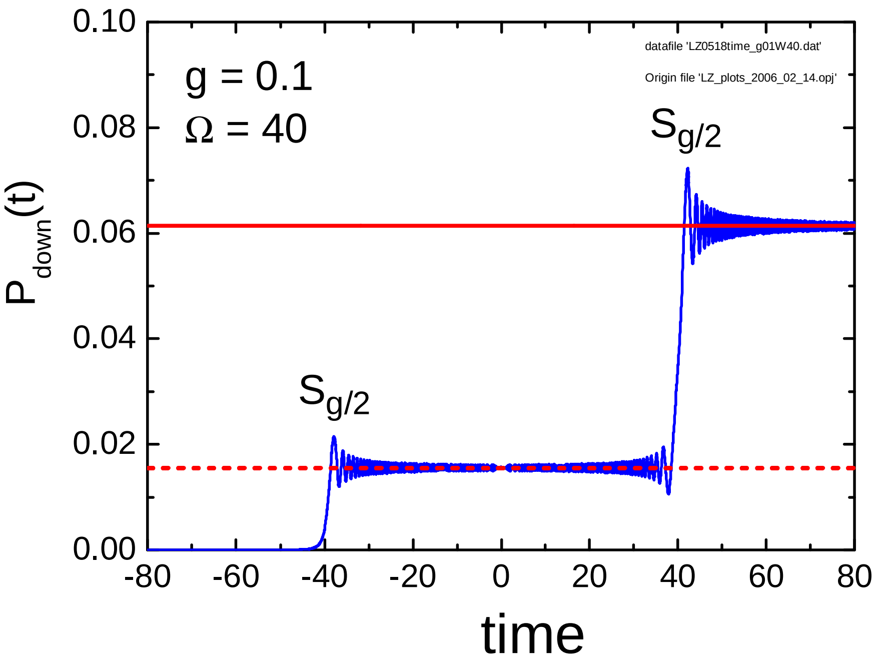
$$H(t) = \frac{vt}{2}\,\boldsymbol{\sigma}_z + g\cos(\Omega t + \phi)\boldsymbol{\sigma}_x$$



Wubs, Saito, Kohler, Kayanuma & Hänggi, N. J. Phys. **7**, 218 (2005).

RWA and transfer-matrix approach

$$\begin{cases} \dot{c}_\uparrow = -i\frac{g}{2\hbar}\left[\, e^{\,iV(t+\hbar\Omega/V)^2} + e^{\,iV(t-\hbar\Omega/V)^2}\,\right] c_\downarrow \\ \dot{c}_\downarrow = -i\frac{g}{2\hbar}\left[e^{-iV(t+\hbar\Omega/V)^2} + e^{-iV(t-\hbar\Omega/V)^2}\right] c_\uparrow \end{cases}$$



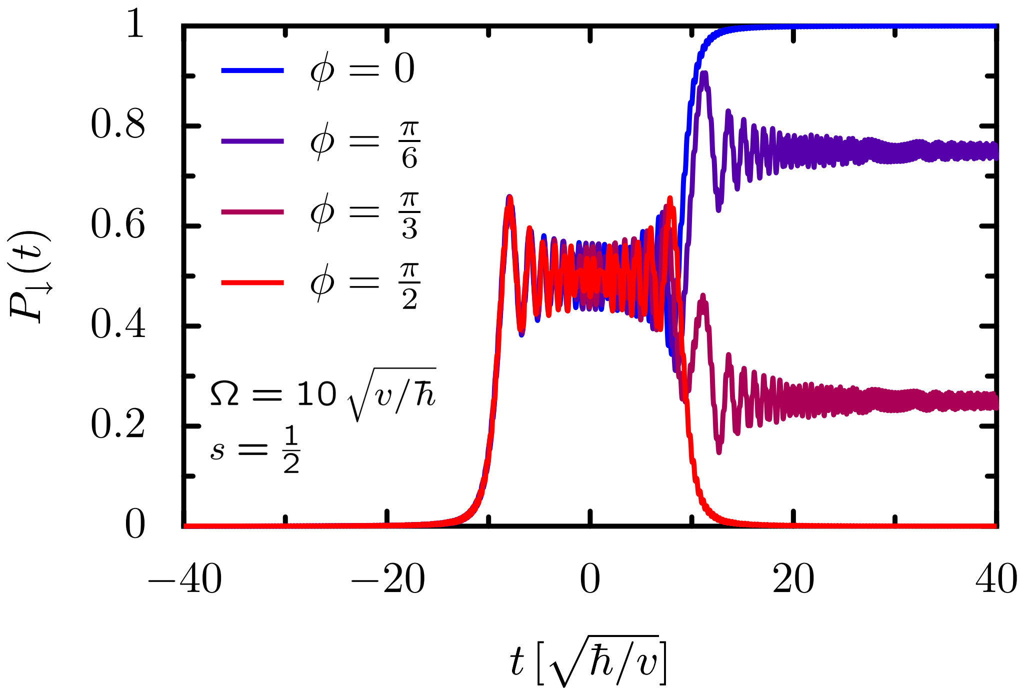
(Oscillator still in coherent state)

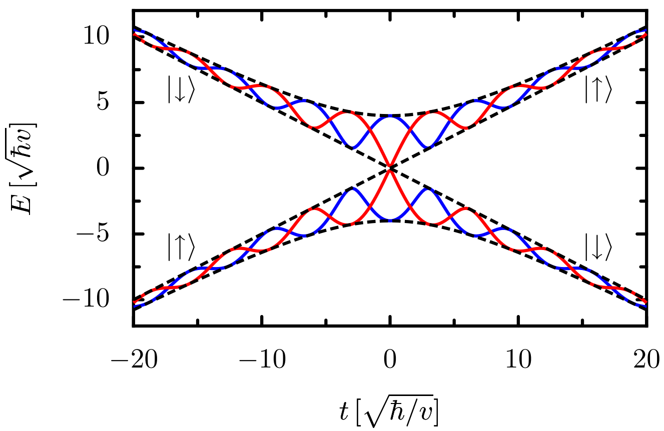
Transfer matrix: $\begin{pmatrix} c_\uparrow(\infty) \\ c_\downarrow(\infty) \end{pmatrix} = \boldsymbol{S}_{g/2} \cdot \boldsymbol{S}_{g/2} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Phase dependence & time-reversal anti-symmetry

$$H_Q(t) = \frac{vt}{2}\,\boldsymbol{\sigma}_z + g\cos(\Omega t + \phi)\boldsymbol{\sigma}_x$$

$$P_\downarrow(\infty) \simeq 4s(1-s)\cos^2\phi, \qquad s \equiv e^{-\pi g^2/2v}$$

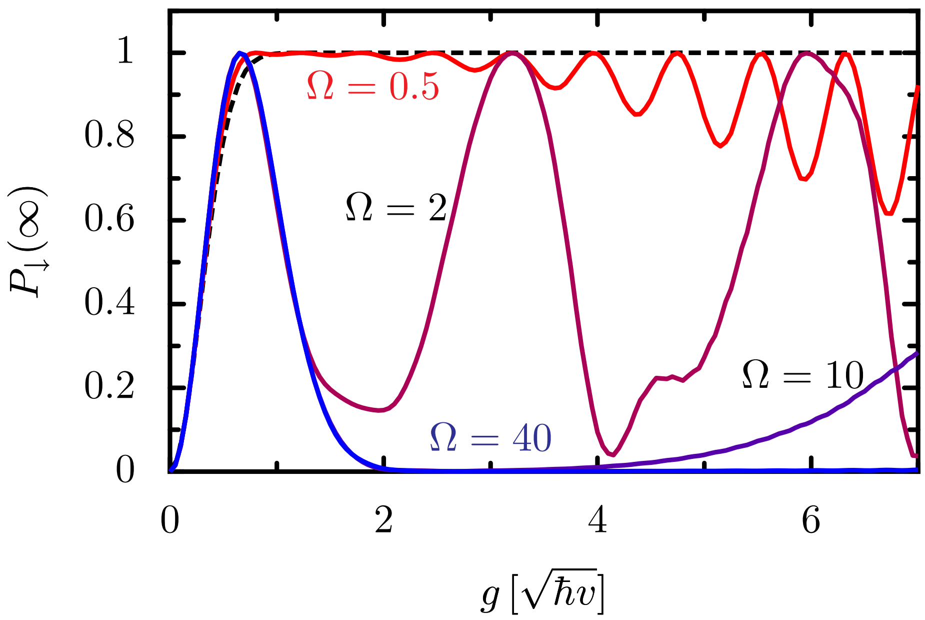


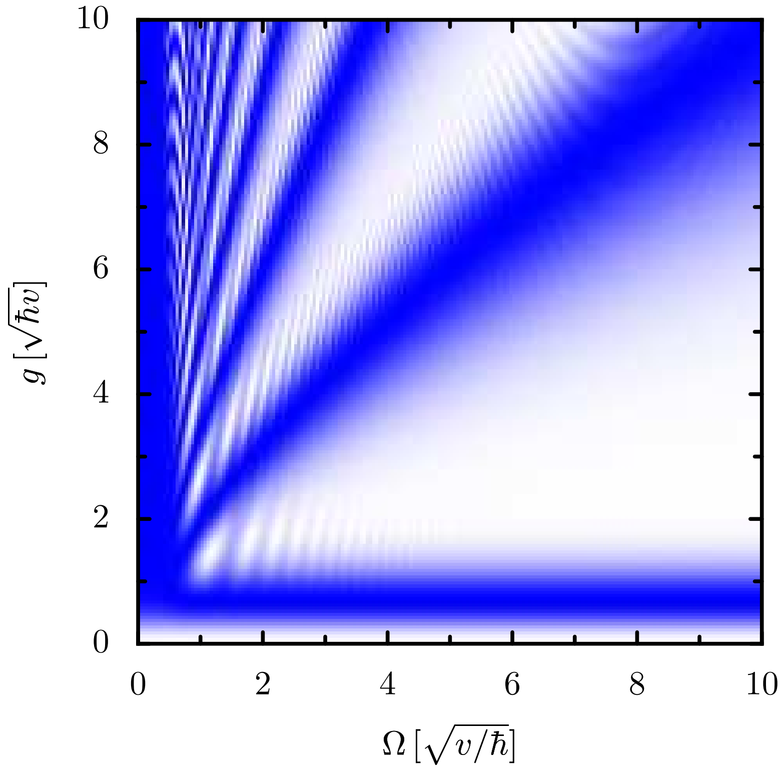


Semiclassical: $P_\downarrow(\infty)$ frequency- and/or phase-dependent

Final transition probabilities ($\phi = 0$)

Semiclassical: $P_\downarrow(\infty)$ frequency dependent



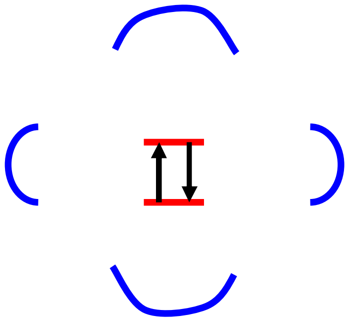


$$\hbar\Omega > g \;\Rightarrow\; P_\downarrow(\infty) \simeq 4s(1-s), \qquad s \equiv e^{-\pi g^2/2\hbar v}$$

Wubs, Saito, Kohler, Kayanuma & Hänggi, N. J. Phys. **7**, 218 (2005).

Back to quantum model: generalizations

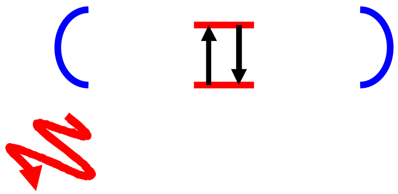
1. Qubit coupled to two or more oscillators



Example: Two-cavity circuit QED

(= proposal by Storcz, Mariantoni, *et al.*)

2. Qubit coupled to infinitely many oscillators

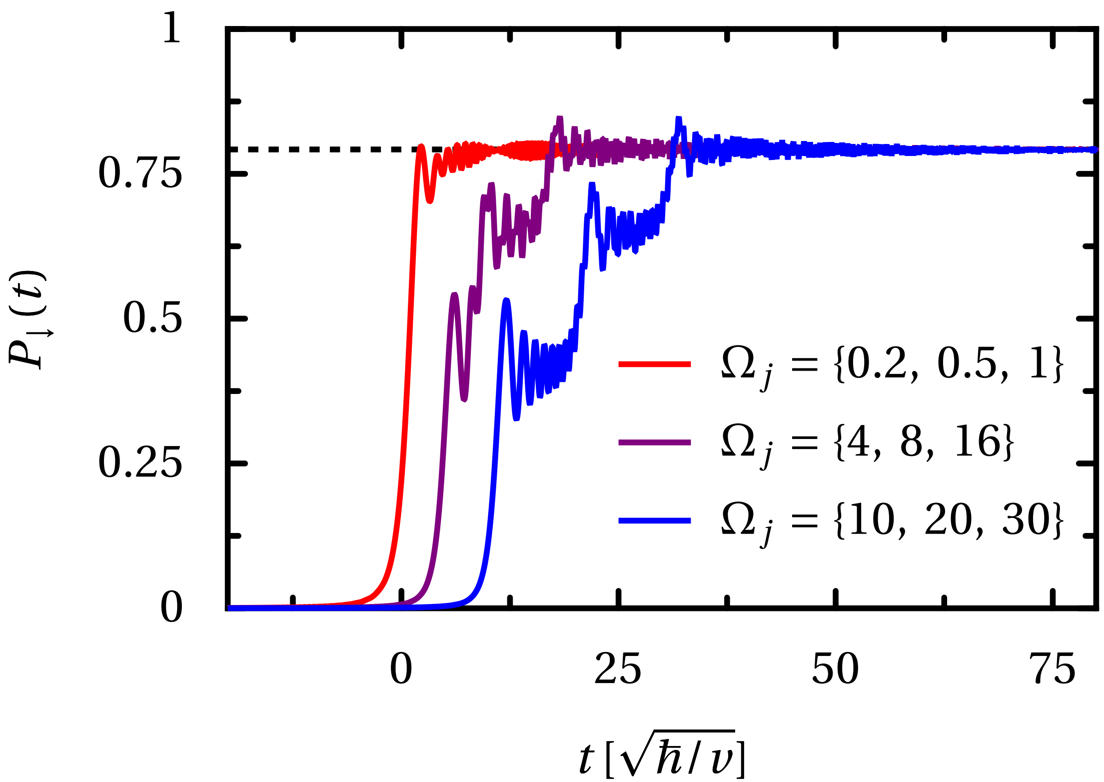


Example: Circuit QED with cavity decay

(= realistic)

Generalization to *N* oscillators:

$$H(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + \sum_{j=1}^{N}\{\gamma_j \boldsymbol{\sigma}_x (b_j + b_j^\dagger) + \hbar\Omega_j\, b_j^\dagger b_j\}$$



$P_\uparrow(\infty)$

Frequency-independent

$$P_\downarrow(\infty) = 1 - \exp\left[-\frac{2\pi}{\hbar v}\sum_{j=1}^{N}\gamma_j^2\right]$$

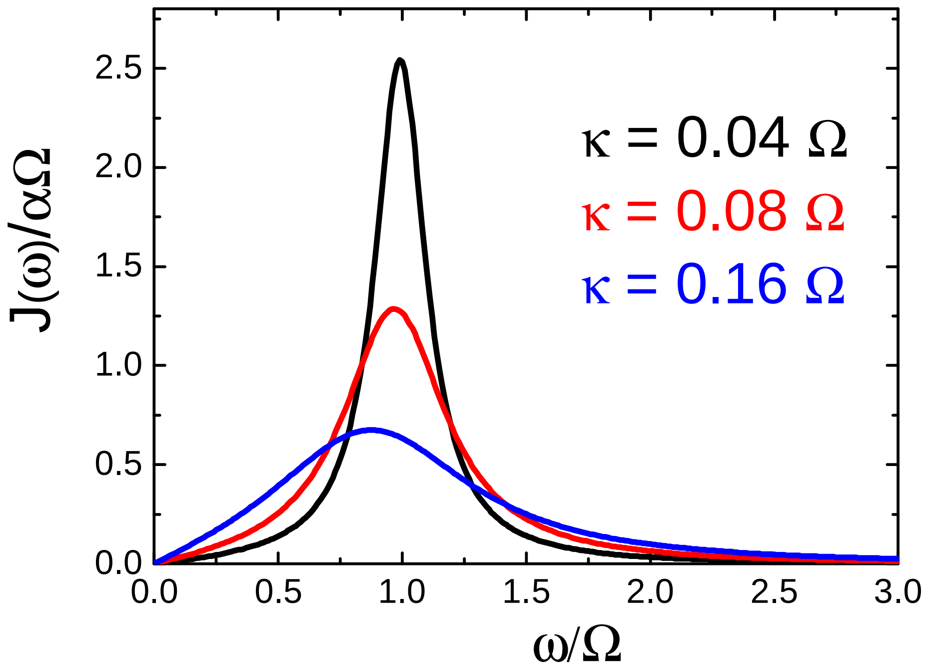
Exact at T=0 (K)

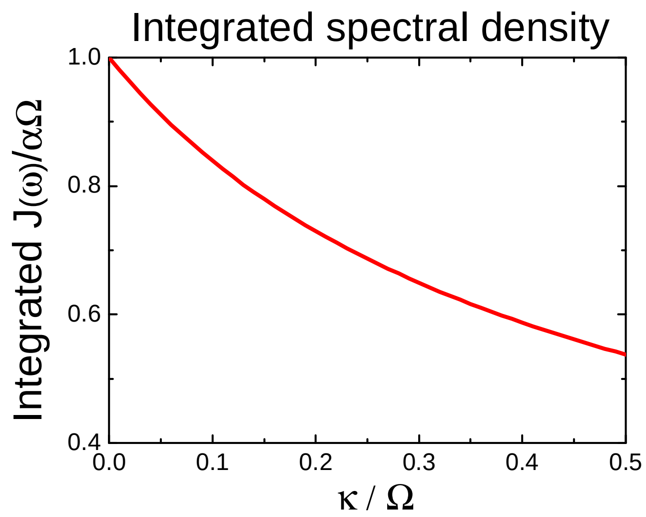
Saito, Wubs, Kohler, Kayanuma, & Hänggi, cond-mat/0603188

Peaked spectral density in circuit QED

$$J(\omega) = \pi \sum_{j=1}^{N} (2\gamma_j/\hbar)^2 \, \delta(\omega - \Omega_j) \Rightarrow$$

$$P_\downarrow(\infty) = 1 - \exp\left[-\frac{\hbar}{2v}\int_0^\infty \mathrm{d}\omega \, J(\omega)\right]$$ Exact at T = 0 (K)





$$J(\omega) = \frac{4\alpha\kappa\Omega^4\omega}{(\Omega^2-\omega^2)^2+(2\pi\kappa\Omega\omega)^2} \qquad (*)$$

(*) Tian *et al*., PRB **65**, 144516 (2002); Van der Wal *et al*., Eur. Phys. J. **31**, 111 (2003).

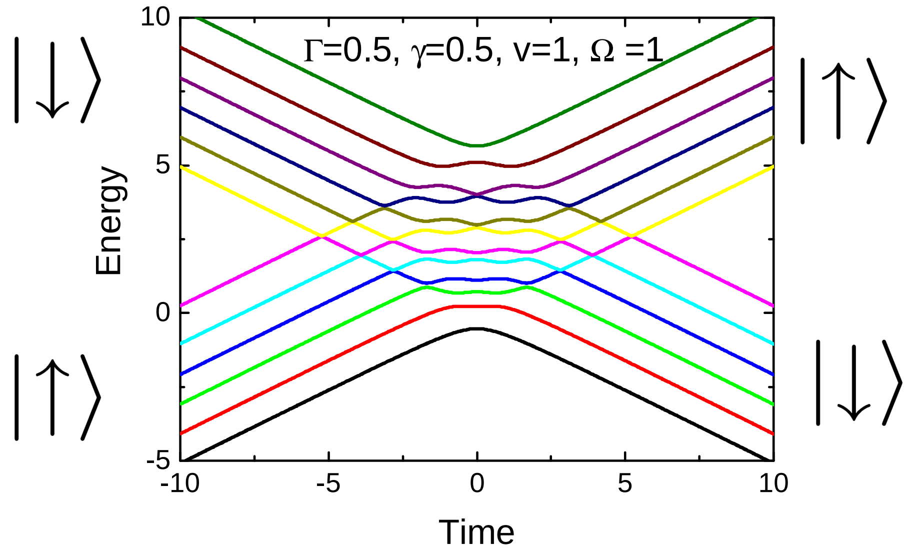
LZ transitions under dephasing

$$H(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + \sum_{j=1}^{N}\{\gamma_j \boldsymbol{\sigma}_x (b_j + b_j^\dagger) + \hbar\Omega_j\, b_j^\dagger b_j\}$$

↓

$$H(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + g\,\boldsymbol{\sigma}_x + \sum_{j=1}^{N}\{\gamma_j\,\boldsymbol{\sigma}_z (b_j + b_j^\dagger) + \hbar\Omega_j\, b_j^\dagger b_j\}$$

↑ **pure dephasing** (*)

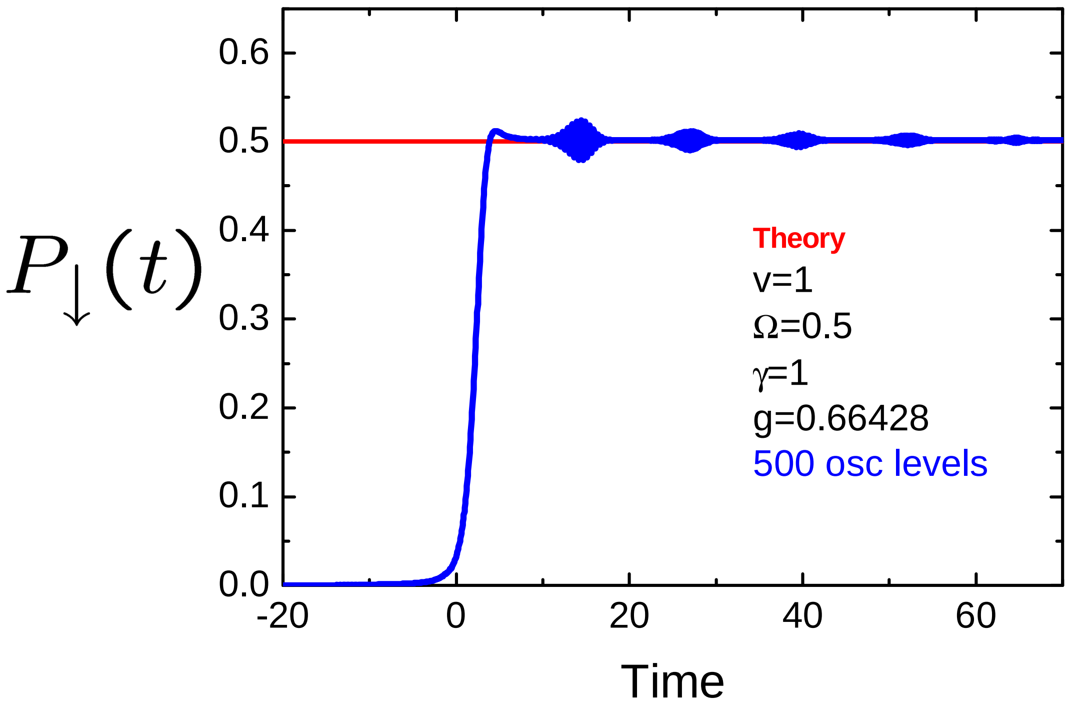


(*) Ao & Rammer ’89, ’91; Kayanuma & Nakayama ‘98

Landau-Zener robust under pure dephasing

$$H(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + g\,\boldsymbol{\sigma}_x + \sum_{j=1}^{N}\{\gamma_j\,\boldsymbol{\sigma}_z(b_j + b_j^\dagger) + \hbar\Omega_j\,b_j^\dagger b_j\}$$

pure dephasing (*)



New exact result:

$$P_\uparrow(\infty) = 1 - \exp\left(-\frac{2\pi g^2}{\hbar v}\right)$$

independent of bath at 0 (K)

(*) Ao & Rammer '89, '91; Kayanuma & Nakayama '98

Conclusions

1. Semiclassical “Landau-Zener meets Rabi problem”: Highly tunable transition probabilities

2. Exact LZ transition probabilities for CCQED

3. LZ cycles for single-photon generation

4. LZ tunneling gauges quantum dissipation

Thanks for your attention!

Sponsored by:

Quantum Information Highway A8

SFB 631

Ministry of Education of Japan