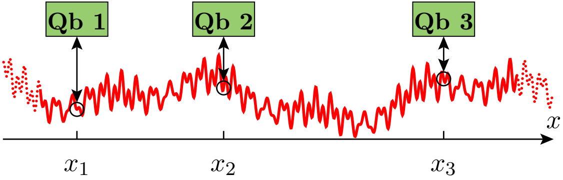
Coherence and entanglement dynamics of exciton qubits

Martijn Wubs, NBI/DTU

AMOLF, Oct. 26, 2009



in collaboration with
R. Doll, S. Kohler, and P. Hänggi (Augsburg)

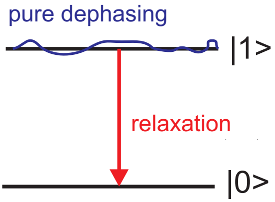
intro: quantum coherence

wave function for isolated systems

$$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

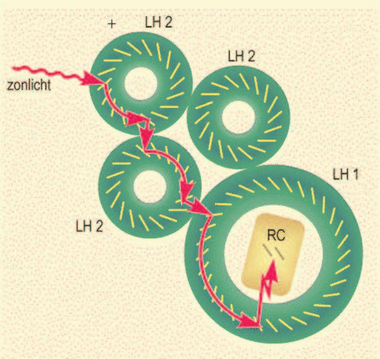
density matrix for open quantum systems

$$\rho(0) = |\psi\rangle\langle\psi| = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| + \frac{1}{2}|0\rangle\langle 1| + \frac{1}{2}|1\rangle\langle 0|$$

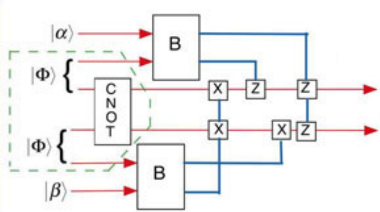


intro: search for quantum coherence

Light harvesting



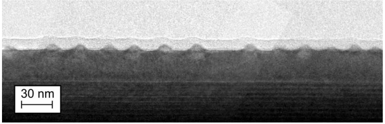
Quantum information processing



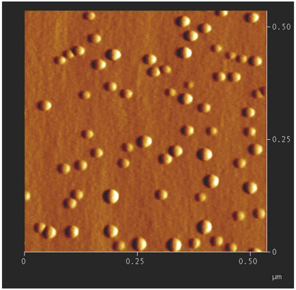
Outline

- recent experiments:
  exciton spectra in 3D and on 1D substrates
- 1 qubit:
  coherence dynamics under dephasing (in 1D, 3D)
- 2 qubits:
  entanglement dynamics, finite separation (in 1D, 3D)
- N qubits:
  benchmarking master equations with exact results
- conclusions

Quantum dots and substrates



from: group Rosenauer, Bremen



quantum dots = artificial atoms

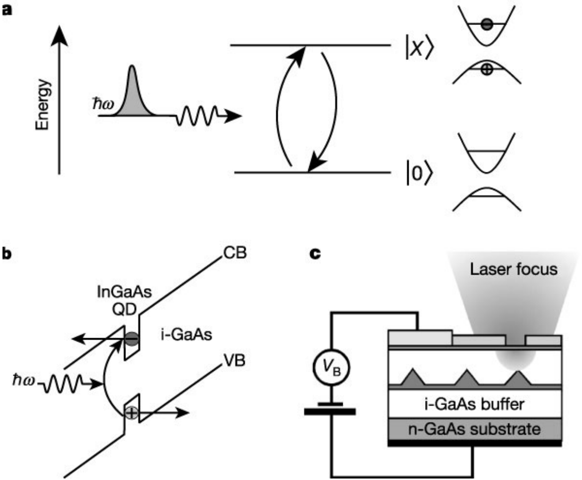
- self-assembled, eg. InGaAs on GaAs
- size 2 – 10nm
  0D: discrete energy levels
- long-lived optical excitations

useful as:

- single-photon sources
- quantum memory? Coherence?

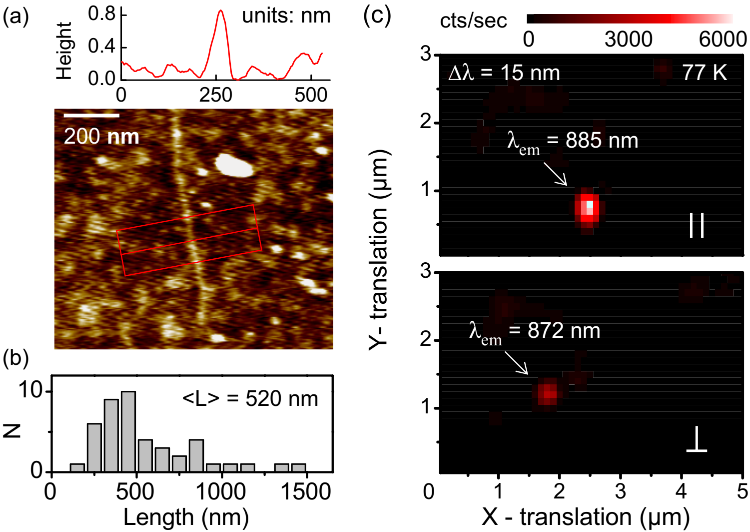
Qubit in a quantum dot

two-level system $|0\rangle$, $|X\rangle$: exciton confined to q-dot



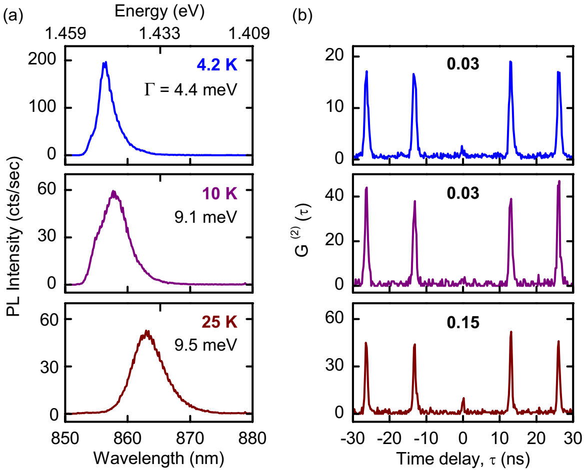
Zrenner *et al.*, Nature (2002).

experiments: exciton qubits in carbon nanotubes



Högele *et al.*, PRL (2008)

experiments: excitons in nanotubes as single-photon emitters



Högele *et al.*, PRL (2008)

"… intrinsic phonon-induced pure dephasing is 2 orders of magnitude larger than the lifetime broadening …", (Galland *et al.* (2008))

1 qubit: modeling pure dephasing due to substrate

- qubit-bath Hamiltonian

$$H = \hbar\Omega\sigma_z + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + H_{\mathrm{q-b}}$$

- interaction

$$H_{\mathrm{q-b}} = \hbar\sigma_z \sum_{\mathbf{k}} \left(g_{\mathbf{k}} b_{\mathbf{k}} + g_{\mathbf{k}}^{*} b_{\mathbf{k}}^{\dagger} \right)$$

- initial state

$$R(0) = \rho_{\mathrm{q}}(0) \otimes \rho_{\mathrm{b}}^{\mathrm{eq.}}$$

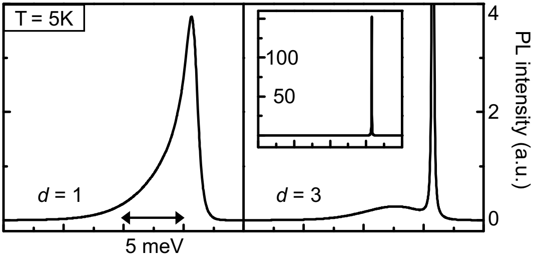
- reduced qubit state

$$\rho_{\mathrm{q}}(t) = \mathrm{Tr}_b[R(t)] = \begin{pmatrix} p_0 & \rho_{01}(0)\,c(t) \\ \rho_{01}^{*}(0)\,c^{*}(t) & 1-p_0 \end{pmatrix},$$

1-qubit coherence function $c(t)$

- deformation-potential coupling to acoustic phonons, $g_{\mathbf{k}} \propto \sqrt{\mathbf{k}}$
- bath spectral density $J(\omega) = \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \delta(\omega - \omega_{\mathbf{k}})$
- linear substrate (1D) → $J(\omega) \propto \omega^1$ ohmic
  bulk substrate (3D) → $J(\omega) \propto \omega^3$ superohmic
- model spectral density with cutoff $\omega_c = v_{\text{phon}} / \ell_{\text{dot}}$

$$J(\omega) = \alpha\omega_c (\omega/\omega_c)^d e^{-\omega/\omega_c}$$



1 qubit: exact coherence for arbitrary substrate dimension d

- reduced qubit state

$$\rho_{\mathrm{q}}(t) = \mathrm{Tr}_b[R(t)] = \begin{pmatrix} p_0 & \rho_{01}(0)\,c(t) \\ \rho_{01}^*(0)\,c^*(t) & 1-p_0 \end{pmatrix},$$

1-qubit coherence function $c(t)$

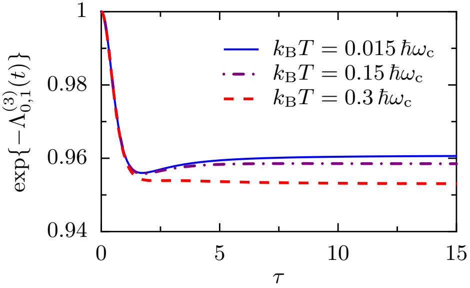
- exact 1-qubit coherence $c_d(t) = \exp[-\lambda_d(t)]$, with (sorry)

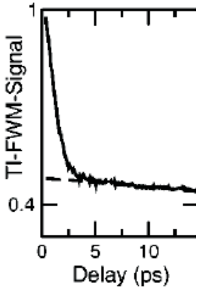
$$\lambda_d(t) = 8\alpha_s(-\theta)^{d-1}\left[F^{d-1}(\theta) - \mathrm{Re}F^{d-1}(\theta[1+i\omega_c t])\right]$$

$$+4\alpha_d\Gamma(d-1)\left(\frac{\cos[(d-1)\arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{(d-1)/2}} - 1\right),$$

- $\Gamma(z)$ is Euler's Gamma function,
- $F(z) = log\Gamma(z)$, and $F^n(z)$ is its n-th derivative

1 qubit: incomplete pure dephasing in bulk substrate



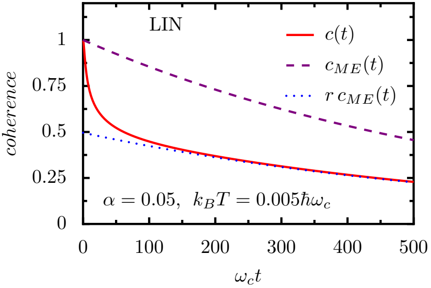


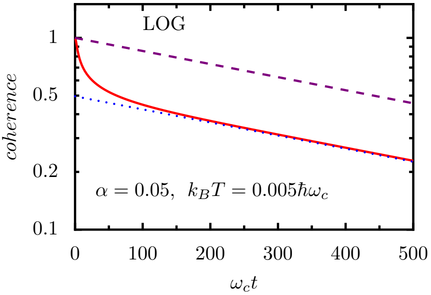
Vagov *et al.* PRB 2004

$$c^{(3)}(\tau \to \infty) \to \exp(-4\alpha) \quad \left(\theta = \frac{k_B T}{\hbar\omega_c} \to 0\right)$$

$\alpha = 0.01$, time scale for fast decay $t_* \simeq \omega_c^{-1}$

1 qubit: complete pure dephasing in a nanotube



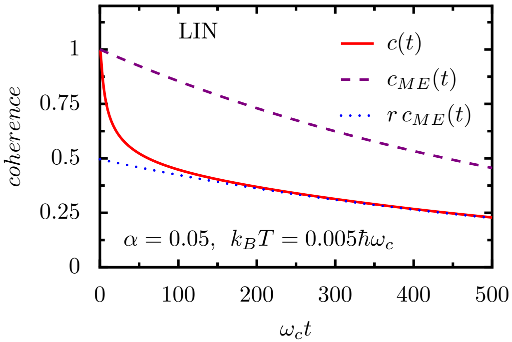


- master equation:
  - $c_{ME}(t) = \exp(-t/T_2)$,
  - $T_2^{-1} = 4\pi\alpha k_B T/\hbar$
- exact coherence :
  - fast initial decay of $c(t)$
  - time scale $\hbar/k_B T$
  - amplitude

$$r = \left[\frac{2\pi\theta^{2\theta-1}}{\Gamma^2(\theta)}\right]^{4\alpha}$$
$$\simeq (2\pi\theta)^{4\alpha} \quad (\theta \ll 1).$$

1 qubit: complete pure dephasing in a nanotube

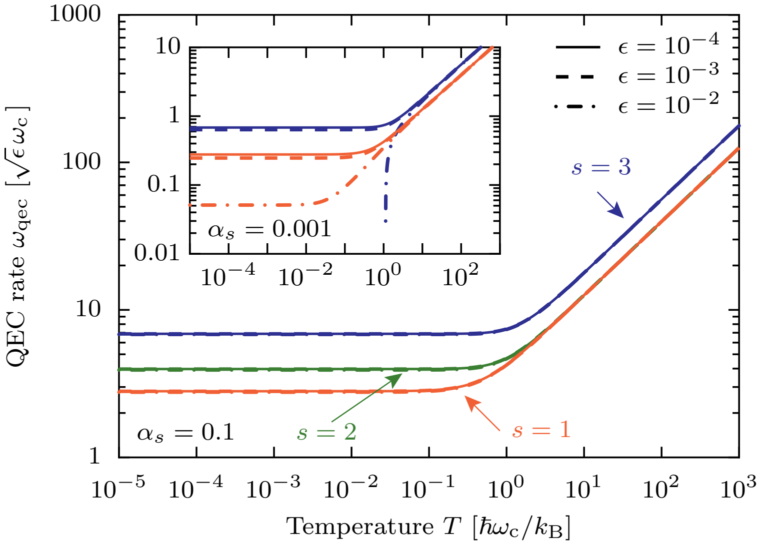
"The non-Markovian nature of this decoherence mechanism may have adverse consequences for applications of one-dimensional systems in quantum information processing." (Galland *et al.*, (2008))



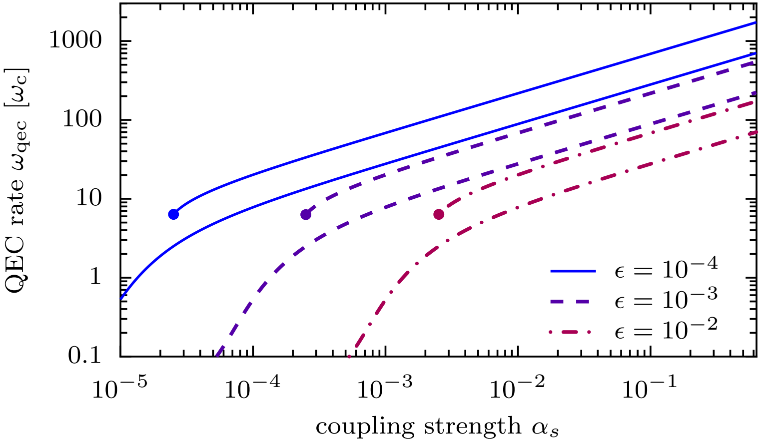
Q: 1D substrates less ideal?

A: compare quantum error correction rates

1 qubit: temperature dependence of QEC rates



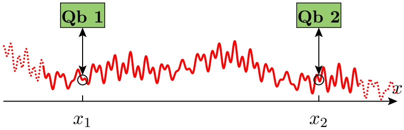
1 qubit: coupling dependence of QEC rates

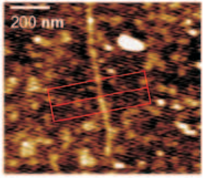


$$k_B T = 0.01 \hbar \omega_c$$

2 qubits: robust Bell states

- Now: 2 qubits at distance x_{12}, coupled to same phonon bath.





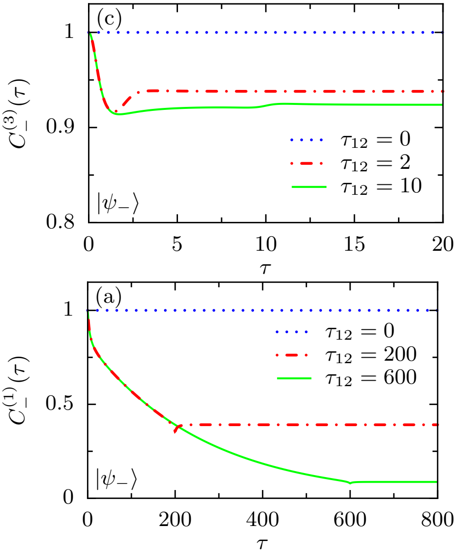
- Known: 2 qubits coupled to same bath at same position:

$$|\psi_{\text{robust}}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

  robust: decoherence-free subspace

- Entanglement measure:
  Concurrence $C[\rho] = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$,
  λ_i are ordered eigenvalues of $\rho\sigma_{1y}\sigma_{2y}\rho^*\sigma_{1y}\sigma_{2y}$.

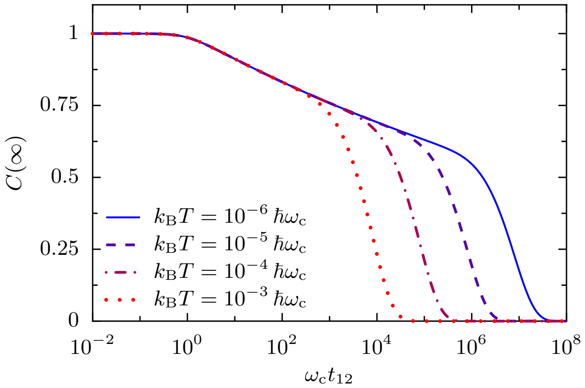
2 qubits: entanglement of robust Bell state



decoherence-poor subspace

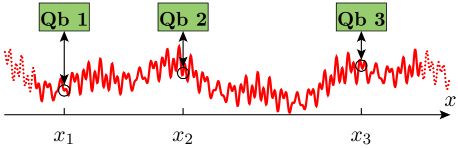
- upper = 3D
  lower = 1D
- coupling $\alpha = 0.01$,
- scaled temp.
  $\theta = k_B T/\hbar\omega_c = 0.015$,
- scaled distance
  $\tau_{12} = \omega_c x_{12}/v_{\text{phon}}$,
- scaled time $\tau = \omega_c t$.

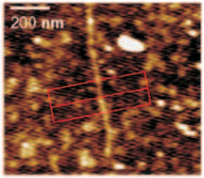
2 qubits: final concurrence of robust Bell state



$$C_-^{(1)}(\tau \to \infty) = (1+\tau_{12}^2)^{4\alpha} \left| \frac{\Gamma[\theta(1-i\tau_{12})]}{\Gamma(\theta)} \right|^{16\alpha} \to \frac{1}{(1+\tau_{12}^2)^{4\alpha}} \quad (\theta\tau_{12} \to 0)$$

N qubits: entanglement dynamics





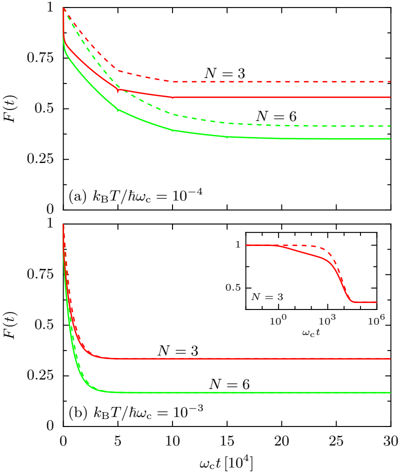
- W states:

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|100\ldots0\rangle + |010\ldots0\rangle + \ldots + |000\ldots1\rangle)$$

  N-qubit generalization of $|W_2\rangle = |\psi_{\text{robust}}\rangle$, robust Bell state

- Entanglement measure for N qubits: problematic
- Here: fidelity $F(t) = \text{Tr}\{\rho(t)\rho(0)\}$

N qubits: fidelity dynamics and master equations



- usual master equation

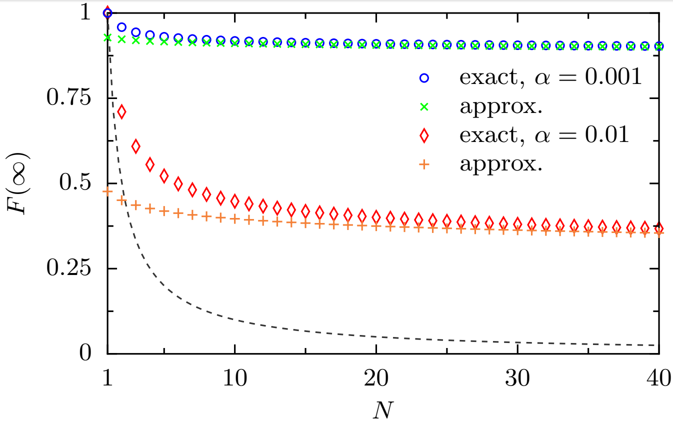
$$F_{ME}(t) = 1 \quad \forall\, t \quad \text{(bad)}$$

- causal master equation

$$\frac{d}{dt}\tilde{\rho}_{jj'}(t) = -\frac{8\alpha\pi k_B T}{\hbar}\left[1 - \Theta(t - t_{jj'})\right]\tilde{\rho}_{jj'}(t)$$

- Two assumptions for QEC violated
  1. exponential decay
  2. qubits in uncorrelated baths

N qubits: scaling of final fidelity



$$F(\infty) \simeq \left[\frac{\Gamma(\frac{1}{2} - 4\alpha)}{\Gamma(\frac{3}{2} - 4\alpha)} - \frac{1}{1-4\alpha} \right] (N\omega_c t_{12})^{-8\alpha} \to (N\omega_c t_{12})^{-8\alpha} \quad (\alpha < 0.005)$$

conclusions

- experiments:
  i. exciton spectra in 3D and on 1D substrates
  ii. non-exponential dephasing challenge for QIP

- 1 qubit:
  i. fast initial dephasing
  ii. stabilization in 3D, exponential decay in 1D
  iii. lowest error correction rates for 1D

- 2 qubits:
  i. entanglement stabilization in 3D
  ii. and in 1D for the robust state

- N qubits:
  i. Incomplete pure dephasing of W states, exact dynamics
  ii. Master equation very inaccurate. We improved it.

thanks to . . .

- Roland Doll
  Sigmund Kohler (Augsburg)
  Peter Hänggi

Thanks for your attention!

1. Two-qubit entanglement, EPL **76**, 547 (2006),
2. Incomplete pure dephasing of N qubits, PRB **76**, 045317 (2007),
3. Quantum error correction rates, EPJ B **68**, 523 (2009).
4. Exact results from approximate master equations, Chem. Phys. **347**, 243 (2008).

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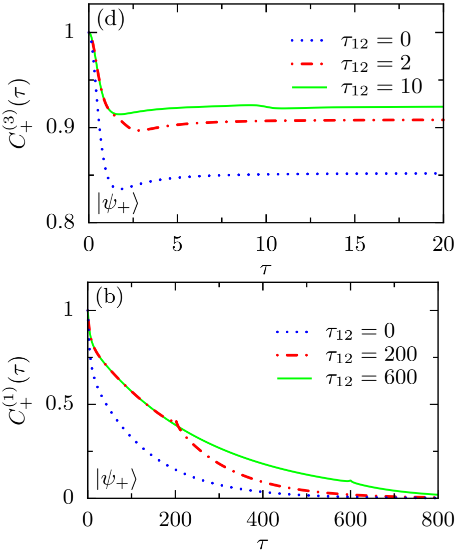
2 qubits: entanglement of fragile Bell state

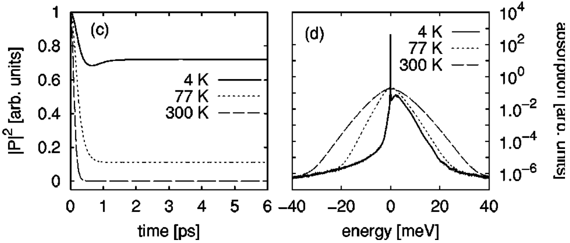


- upper = 3D, superohmic
  lower = 1D, ohmic
- coupling $\alpha = 0.01$,
  scaled temp.
  $\theta = k_B T/\hbar\omega_c = 0.015$,
  scaled distance
  $\tau_{12} = \omega_c x_{12}/v_{\mathrm{phon}}$,
  scaled time $\tau = \omega_c t$.
- general identity

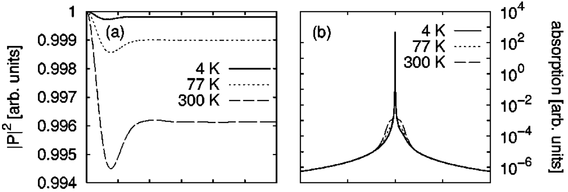
$$C_+^{(d)}(t)\, C_-^{(d)}(t) = |c^{(d)}(t)|^4$$

Deformation-potential coupling dominates

Pure dephasing due to deformation-potential coupling:

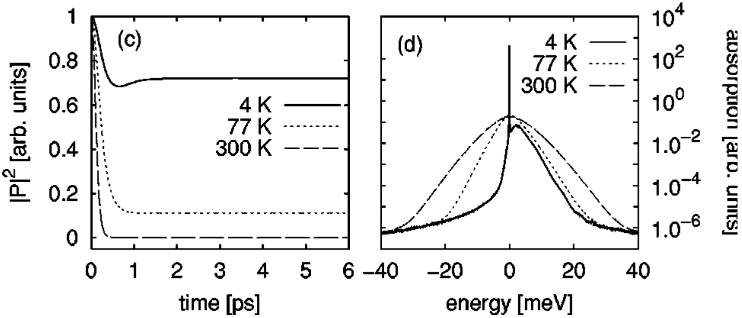


Pure dephasing due to piezo-electric coupling:



Krummheuer *et al.*, PRB (2002)

1 qubit: Non-Markovian dephasing and lineshape



Krummheuer *et al.* (2002)