Scattering Induced Quantum Interference of Multiple Quantum Optical States

Johan R. Ott, M. Wubs, N. Asger Mortensen and Peter Lodahl

Technical University of Denmark, Institute of Photonics Engineering, Bygn. 343, 2800 Kgs. Lyngby, Denmark

Abstract. Using a discrete mode theory for propagation of quantum optical states, we investigate the consequences of multiple scattering on the degree of quadrature entanglement and quantum interference. We report that entangled states can be created by multiple-scattering. We furthermore show that quantum interference induced by the transmission of quantized light through a multiple-scattering medium will persist even after averaging over an ensemble of scattering samples.

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INTRODUCTION

The propagation of waves in complex scattering media can result in many fascinating phenomena such as Anderson localization [1], enhanced coherent back scattering [2, 3], and universal conductance fluctuations [4]. Recently it was shown experimentally that light-matter interaction is strongly enhanced in disordered photonic crystal waveguides, enabling cavity quantum electrodynamics with Anderson-localized modes [5]. This potentially makes disordered structures useful in future quantum information processing.

Optical quantum information processing schemes rely on interference among multiple independent quantum states, i.e. quantum interference, to generate quantum correlations and entanglement. The possibility of using multiple scattering media to interfere independent quantum states appears appealing since it is inherently scalable to multiple input states. To this end mesoscopic quantum interference effects are required since they would persist even after averaging over all realization of disorder, thereby providing robust and predictable quantum correlations.

Here we report on the effects of quantum interference induced by combining an arbitrary number of quantum optical states in a random multiple scattering medium [6]. We identify the role of quantum interference on the degree of photon correlations between two transmission paths through the medium and the degree of continuous variable entanglement. The investigation is performed using a scattering matrix formalism and for the statistical properties we rely on random matrix theory.

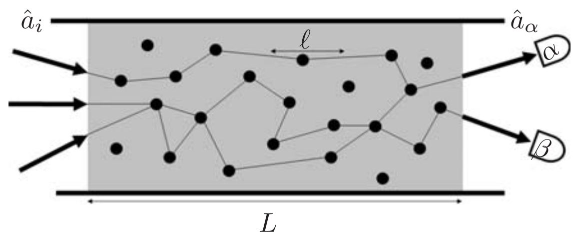

FIGURE 1. Sketch of propagation through a complex scattering structure of length L and transport mean free path ℓ. Quantized light is incident to the left and the correlations between two output modes on the right are analyzed. The operators $\hat{a}_i$ and $\hat{a}_\alpha$ corresponds to the annihilation operators of modes i and α, where Roman and Greek subscripts denote input and output modes respectively. The correlations between two different output modes α and β are analyzed.

MODEL AND QUANTUM MEASURES

We describe propagation of quantized light through a linear, elastic, multiple scattering medium of length L and transport mean free path ℓ, see Fig. 1. To model this we use a quasi one-dimensional discrete mode theory which is known also to describe well the propagation in slab geometries [7] and has previously been used to describe propagation of quantum optical states in complex structures [8, 9, 10]. Experimentally such systems have been realized e.g. in titania powder samples [11, 12] or disordered photonic crystal waveguides [5].

We apply the scattering matrix for the propagation of light and use random matrix theory on the scattering elements. Thus we relate the photon annihilation operators $\hat{a}_\alpha$ and $\hat{a}_i$ of output and input modes through $\hat{a}_\alpha = \sum_i t_{\alpha i} \hat{a}_i$, where the summation is over all N possible input modes at each end of the waveguide and $t_{\alpha i}$ denotes the complex scattering matrix element.

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As a measure of quantum interference we use the normalized photon number correlations between two different output modes, i.e. the 2-channel correlation function

$$C_{\alpha,\beta} = \frac{\Delta\hat{n}_\alpha\hat{n}_\beta}{\langle\hat{n}_\alpha\rangle\langle\hat{n}_\beta\rangle}, \tag{1}$$

where

$$\Delta\hat{n}_\alpha\hat{n}_\beta = \langle\hat{n}_\alpha\hat{n}_\beta\rangle - \langle\hat{n}_\alpha\rangle\langle\hat{n}_\beta\rangle, \tag{2}$$

$\langle:\rangle$ denote the quantum mechanical expectation value, the indices α and β denote different output modes and $\hat{n}_\alpha = \hat{a}^\dagger_\alpha\hat{a}_\alpha$ is the photon number operator where $\hat{a}^\dagger_\alpha$ ($\hat{a}_\alpha$) is the photon creation (annihilation) operator of mode α. This quantity describes the conditional probability of detecting photons in mode α given a measurement of photons in mode β or vice versa.

Furthermore as a measure for entanglement we calculate the degree of quadrature entanglement [13, 14, 15]

$$\varepsilon_{\alpha,\beta} = \Delta(\hat{X}_\alpha - \hat{X}_\beta)^2\Delta(\hat{Y}_\alpha + \hat{Y}_\beta)^2, \tag{3}$$

where $\hat{X}_\alpha = \frac{1}{\sqrt{2}}(\hat{a}^\dagger_\alpha + \hat{a}_\alpha)$ and $\hat{Y}_\alpha = \frac{i}{\sqrt{2}}(\hat{a}^\dagger_\alpha - \hat{a}_\alpha)$ are the quadrature operators. The degree of quadrature entanglement is a measure of the separability of the states in two modes and the value $\varepsilon_{\alpha,\beta} < 1$ implies that the states are inseparable, i.e. entangled [13, 14, 15].

ENSEMBLE AVERAGE

Since multiple scattering is a random process we need to describe the average values for an ensemble of scattering media having the same scattering properties. From random matrix theory one can relate the ensemble averages of products of amplitude transmission coefficients to the classical short- and long-range correlation functions, C_1 and C_2, and the average conductance, g as [16]

$$\overline{t^*_{\alpha i}t_{\alpha j}} = \tau\delta_{ij}, \tag{4}$$

$$\overline{t^*_{\alpha i}t^*_{\beta j}t_{\beta k}t_{\alpha l}} = \tau^2\left(C_1\delta_{il}\delta_{jk} + C_2\delta_{ik}\delta_{jl}\right), \tag{5}$$

with $\tau = gN$ being the average single channel intensity transmission coefficient and the bar denotes ensemble averaging. These expressions are non-perturbative and valid in the entire mesoscopic regime [16]. From the values of C_2 and the normalized average conductance g, we define the transitions from the quasi-ballistic to the weakly disordered regime (C_2=0) and from the weakly disordered to the localized regime (g= 1). The mesoscopic regime is defined as the regime in which two speckle spots are correlated after ensemble averaging ($C_2 > 0$). The values of C_1 and C_2 depend both on the number of modes N and the degree of disorder, which is contained in $s = L/\ell$ and g^{-1}. The dependence of C_1, C_2, and g on the number of modes N is non-trivial, but the values of C_1 and C_2 are independent of N on the transition between the quasi-ballistic and the mesoscopic regimes, $s\approx 2$, and tend toward the same value far into the localized regime, $g^{-1} \ll 1$ [17]. The values of C_1 and C_2 have qualitatively the same behaviors versus disorder in the various regimes for different N.

Let us now define the ensemble averaged 2-channel correlation function as the separate ensemble average of the nominator and denominator respectively, i.e.

$$\overline{C}_{\alpha\beta} = \frac{\overline{\Delta\hat{n}_\alpha\hat{n}_\beta}}{\overline{\langle\hat{n}_\alpha\rangle\langle\hat{n}_\beta\rangle}}. \tag{6}$$

Notice that this definition trivially yields zero for incident coherent states corresponding to classical light states independent of the amount of scattering.

Using the above described averaging approach the normalized disorder averaged photon number correlations, $\overline{C}_{\alpha,\beta}$, and the degree of quadrature entanglement, $\overline{\varepsilon}_{\alpha,\beta}$, are related to the classical short- and long-range correlation functions, C_1 and C_2. Thereby the ensemble-averaged 2-channel correlation function is found to be [6]

$$\overline{C}_{\alpha\beta} = \frac{(C_1+C_2)\left[\left(\sum_i\langle\hat{n}_i\rangle\right)^2 + \sum_i(\Delta\hat{n}_i^2 - \langle\hat{n}_i\rangle)\right]}{C_1\left(\sum_i\langle\hat{n}_i\rangle\right)^2 + C_2\left(\sum_i\langle\hat{n}_i\rangle^2 + 2\sum_{i,j>i}|\langle\hat{a}^\dagger_i\hat{a}_j\rangle|^2\right)} - 1, \tag{7}$$

and the ensemble averaged QVP is

$$\overline{\varepsilon}_{\alpha\beta} = 1 + 4\tau\sum_i\Delta\hat{a}^\dagger_i\hat{a}_i + 4\tau^2\left[C_1\left(\sum_i\Delta\hat{a}^\dagger_i\hat{a}_i\right)^2 + C_2\sum_{i,j}\Delta\hat{a}^\dagger_i\hat{a}_j\Delta\hat{a}^\dagger_j\hat{a}_i\right]. \tag{8}$$

For different incident Fock (i.e photon) states the effect of quantum interference is found to survive ensemble averaging and manifest itself as increased photon correlations when the quantum states are incident from more than one direction. The quantum interference effect increases with the amount of scattering and can even become positive, see Fig. 2. This highly counterintuitive result signify that having detected a photon in one output mode increase the possibility of detecting one in another output direction even though the measurement has removed at least one photon from the system. It is furthermore demonstrated that it is possible to use a multiple scattering medium to induce entanglement between output modes, while this entanglement does not survive ensemble averaging [6] (not shown in the figure here).

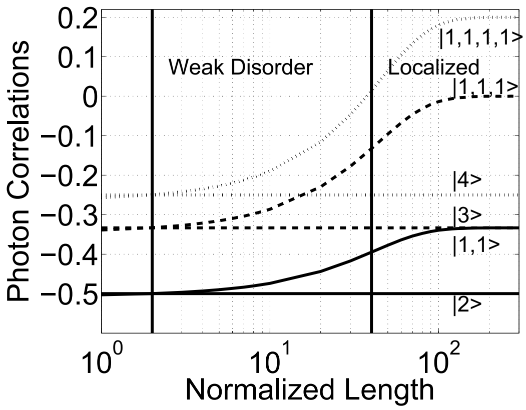

FIGURE 2. The ensemble averaged 2-channel photon correlations, $\overline{C}_{\alpha\beta}$ versus $s{=}L/\ell$ for $N = 50$. Solid curves show $\overline{C}_{\alpha\beta}$ for Fock input states with a total of two photons incident from either one, $|2>$, or two, $|1,1>$, directions, dashed curves are for three photons incident from either one, $|3>$, or three, $|1,1,1>$, directions and the dotted curves are for four photons from either one, $|4>$, or four, $|1,1,1,1>$, incident directions. The difference in $\overline{C}_{\alpha\beta}$ between having one and more input states is due to quantum interference. The symbols $\circ$, $\times$, and $*$ on the abscissa correspond to the experimental structures studied in Refs. [12], [11], and [18] respectively.

In Fig. 2 we indicate the position of three existing multiple scattering structures from the literature, where the number of modes N has been scaled to match the value used in the calculations. Ref. [12] concerns transmission through two scattering surfaces, which mimic a multiple scattering medium. This corresponds to the diffusive limit where quantum interference will be present in the speckle pattern, but not survive ensemble averaging. In Ref. [11] a titania powder is used with sample length $L{=}20\,\mu$m and transport mean free path $\ell{\approx}0.9\,\mu$m, which corresponds to the mesoscopic regime. Such samples support a large number of modes ($N{>}10^3$) and thus $g{\gg}1$, which means that this type of sample is in the weakly disordered regime where quantum interference effects are modest, cf. Fig. 2. This illustrates the importance of using multiple scattering samples supporting only few modes in order to observe quantum interference. A disordered multimode photonic crystal waveguide is exactly such a system and for $N{\approx}5$ and assuming typical experimental parameters of $\ell{\approx}20\,\mu$m and $L{=}100\,\mu$m [18] gives rise to sizeable quantum interference effects that will be observable in an experiment, cf. Fig. 2.

In conclusion, we have shown that the complex process of multiple scattering induces quantum interference that persists after ensemble-averaging. Furthermore our work shows that creation of quadrature entanglement by multiple scattering of squeezed quantum states is possible while such quantum correlations do not persist after ensemble averaging. It would be interesting to investigate the possibility of entanglement creation in the backscattering direction since fascinating correlation phenomena are known to occur here also for classical measurements [2, 3].

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