BATH-INDEPENDENT TRANSITION PROBABILITIES IN THE DISSIPATIVE LANDAU-ZENER PROBLEM

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We study Landau-Zener transitions of a two-level system that is coupled to a quantum heat bath at zero temperature. In particular, we reveal that for a whole class of models, the probability for a nonadiabatic transition is bath-independent.

Keywords: Nonadiabatic transitions; quantum dissipation.

1. Introduction

Nonadiabatic transitions at avoided level crossings play an essential role in numerous dynamical phenomena in physics and chemistry. They have been studied both theoretically and experimentally in various contexts like spin-flip processes in nano-scale magnets,[1,2] molecular collisions,[3] optical systems,[4] quantum-dot arrays,[5] Bose-Einstein condensates,[6] and recently also in quantum information processing.[7–10]

The “standard” Landau-Zener problem describes the ideal situation in which the dynamics is restricted to two levels that are coupled by a constant tunnel matrix element and cross at a constant velocity. The quantity of primary interest is the probability that finally the system ends up in the one or the other of the two states. This classic problem was solved independently by several authors in 1932.[11–14]

In an experiment, the two-level system will be influenced by its environment, which may affect the quantum phase of the superposition, alter the

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effective interaction between the levels, or may cause spontaneous decay. The environment of a quantum system can often be described as a bath of harmonic oscillators.[15–19] In some situations, it is known that the dominant environmental effects can best be modelled as a spin bath instead,[20–22] for example for molecular magnets[1] and for Josephson phase qubits.[7]

In the presence of a heat bath, the Landau-Zener dynamics will sensitively depend on the qubit operator to which the bath couples.[23,24] Ao and Rammer[25] studied Landau-Zener transitions for the special case in which an ohmic heat bath couples to the same operator as the driving and derived the transition probabilities in the limit of high and of low temperatures. In the limits of very fast and very slow sweeps at zero temperature, they found that the transition probability is the same as in the absence of the heat bath, as was confirmed by numerical studies.[26,27]

This zero-temperature result was recently proven to hold exactly for *arbitrary* Landau-Zener sweep speeds, as a special case of an exact expression for arbitrary qubit-bath couplings and spectral densities.[23] An exact solution is also possible if the decoherence stems from the coupling of the system to a spin bath.[24]

2. The Dissipative Landau-Zener Problem

The dissipative Landau-Zener problem, is specified by the system-bath Hamiltonian

$$H(t) = H_{\rm LZ}(t) + H_{\text{q-env}} + H_{\rm env}, \tag{1}$$

where $H_{\rm env}$ and $H_{\text{q-env}}$ describe the environment and its coupling to the two-level system, henceforth termed qubit. The time-dependent qubit Hamiltonian reads

$$H_{\rm LZ}(t) = \frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x, \tag{2}$$

which defines the "standard" Landau-Zener problem. The adiabatic energies, i.e. the eigenstates of $H_{\rm LZ}(t)$ form at time $t = 0$ an avoided crossing between the diabatic states $|{\uparrow}\rangle$ and $|{\downarrow}\rangle$. The latter are the eigenstates of $H_{\rm LZ}(t)$ at large times.

If the qubits starts at time $t = -\infty$ in state $|{\uparrow}\rangle$, one finds that finally at time $t = \infty$, the qubit will be in state $|{\uparrow}\rangle$ with a probability given by the classic expression[11–13]

$$P_{\uparrow\to\uparrow} = \exp\Big(-\frac{\pi\Delta^2}{2\hbar v}\Big). \tag{3}$$

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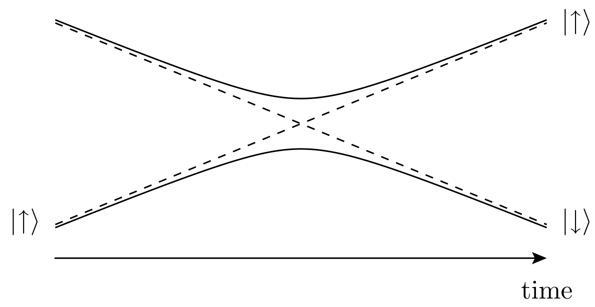

Fig. 1. Adiabatic (solid) and diabatic (dashed) energy levels of "standard" Landau-Zener Hamiltonian (2).

In general, this transition probability is modified by a coupling to the environment[23,24] for which we assume the form $H_{\text{q-env}} = \vec{\sigma} \cdot \vec{n}\xi$, where $\vec{n}$ determines the "direction" of the coupling and ξ is a collective coordinate of the bath. In the following, we explore for which types of coupling the opposite holds, namely that $P_{\uparrow\to\uparrow}$ still is given by Eq. (3), despite the coupling to the bath.

3. Multi-Level Landau-Zener Dynamics

The two-level Landau-Zener problem defined by the Hamiltonian (1) can be mapped to the multi-level Landau-Zener problem sketchetd in Fig. 2 which has been solved in Ref. 24. It is defined by the Hamiltonian

$$\begin{aligned} H(t) = &\sum_a (\varepsilon_a + \frac{vt}{2})|a\rangle\langle a| + \sum_b \big(\varepsilon_b - \frac{vt}{2})|b\rangle\langle b| \\ &+ \sum_{a,b} (X_{ab}|a\rangle\langle b| + X_{ab}^*|b\rangle\langle a|\big), \end{aligned} \tag{4}$$

which describes a group of levels $|a\rangle$ whose energy increases linearly in time, while the energy of the levels $|b\rangle$ deceases. In the limit $t \to \pm\infty$, the states $|a\rangle$, $|b\rangle$ become eigenstates of the Hamiltonian (4), which means that they represent the diabatic eigenstates. The off-diagonal part of the Hamiltonian is such that it only couples states of different groups while states within one group are uncoupled.

If now the system starts in any non-degenerate state $|a\rangle$, one can derive for the transition to a state $|a'\rangle$ the following two statements:[24] First, the

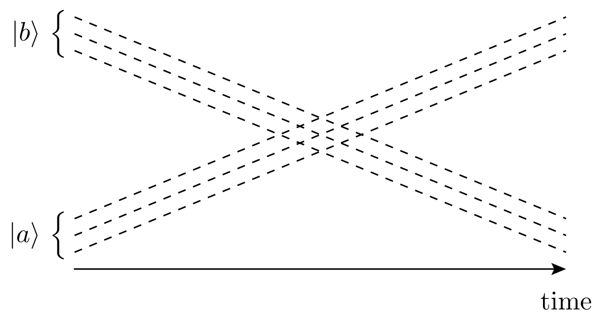

Fig. 2. Diabatic levels of the multi-level Hamiltonian to which we map the dissipative Landau-Zener problem.

probability to end up in the initial state is given by

$$P_{a\to a} = \exp\Big(-\frac{2\pi\langle a|X^2|a\rangle}{\hbar v}\Big), \tag{5}$$

while, second, all a-states with higher energy are finally not populated, i.e.

$$P_{a\to a'} = 0 \quad \text{for } \varepsilon_{a'} > \varepsilon_a. \tag{6}$$

If in particular, the initial state is the a-state with the lowest energy, relation (6) implies that $1 - P_{a\to a}$ denotes the probability to end up in any state of group b.

4. Bath-Independent Landau-Zener Probability

In order to make use of the results of the last section, we have to identify the diabatic states of the dissipative Landau-Zener Hamiltonian (1). Since at large times, the time-dependent part of the Hamiltonian dominates, the diabatic qubit states are the eigenstates of σ_z, $|{\uparrow}\rangle$ and $|{\downarrow}\rangle$. The bath states corresponding to $|{\uparrow}\rangle$ are determined by the Hamiltonian

$$H_{\text{env}\uparrow} = \langle\uparrow|\vec{n}\cdot\vec{\sigma}|\uparrow\rangle\xi + H_\text{env} = n_z\xi + H_\text{env} \tag{7}$$

and will be denoted by $|\nu_+\rangle$ with $|0_+\rangle$ being the ground state. Thus the states $|{\uparrow},\nu_+\rangle$ correspond to group a while the accordingly defined states $|{\downarrow},\nu_-\rangle$ form group b. The coupling operator X then becomes

$$X = \frac{\Delta}{2}\sigma_x + (n_x\sigma_x + n_y\sigma_y)\xi. \tag{8}$$

Note that $n_z\sigma_z\xi$ does not couple states from different groups and, thus, is not contained in X.

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At zero temperature, the natural initial state of the qubit coupled to the bath is the diabatic state $|\uparrow, 0_+\rangle$, which is the ground state in the limit $t \to -\infty$. Since Eq. (6) implies that all states $|\uparrow, \nu_+\rangle$ with $|\nu_+\rangle \neq |0_+\rangle$ will finally be unoccupied, we find

$$P_{\uparrow\to\uparrow} = \sum_{\nu_+} P_{\uparrow,0_+\to\uparrow,\nu_+} = \exp\Big(-\frac{2\pi\langle\uparrow,0_+|X^2|\uparrow,0_+\rangle}{\hbar v}\Big), \qquad (9)$$

where the coupling operator X is given by Eq. (8).

A particular case is now $\vec{n} = \vec{e}_z$ for which $X = \frac{\Delta}{2}\sigma_x$, such that the bath couples only via the Pauli matrix that determines the diabatic states. Then $\langle\uparrow, \nu_+|(\frac{\Delta}{2}\sigma_x)^2|\uparrow, \nu_+\rangle = \Delta^2/4$ and, consequently, the transition probability for a diabatic transition in the presence of a heat bath at zero temperature, Eq. (3), becomes identical to the "standard" Landau-Zener result (9). This result holds true at zero temperature whenever the bath couples to the qubit via σ_z, irrespective of the nature and the spectral density of the bath.

An important experimentally relevant case for which this prediction applies is the measurement of tiny tunnel splittings Δ in nanomagnets[1] for which laboratory experience tells us that at temperatures well below 1K, Landau-Zener tunneling is robust against dephasing. Recent theories[2] for multiple Landau-Zener transitions in such systems presumed that during the individual Landau-Zener transitions, dephasing does not play a role, which is in accordance with our results. Our results show that these theories should be more widely applicable than guessed previously.

5. Conclusions

We have investigated the dissipative Landau-Zener problem for a qubit with a qubit-bath coupling that commutes with the time-dependent part of the qubit Hamiltonian. For large time, this bath coupling causes pure dephasing, while it can induce spin flips at the center of the avoided crossing of the adiabatic levels. As a central result, we have shown that at zero temperature, the Landau-Zener transition probability is dissipation independent. This result holds true for all quantum heat baths with a non-degenerate ground state.

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